MA/MSCMT-05

December - Examination 2018

M.A. / MSc. (Previous) Mathematics Examination

Mechanics

Paper - MA/MSCMT-05

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Type Questions)

Note: Section 'A' contains Very short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Write expressions for moment of inertia (M. I.) of a circular disc of mass M and radius a about its diameter.

(ii) State principle of conservation of linear momentum.

(iii) Define centre of Suspension.

(iv) Explain conservative system.

(v) State principle of least action.

(vi) Explain turbulent flow.

(vii) Write Euler's dynamical equation in vector form.

(viii)Define strength of a source.

Section - B **4 × 8 = 32**

(Short Answer Type Questions)

Note: Section 'B' contain 08 short Answer Type Questions. Examinees will have to answer any four (4) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Show that in the free motion of body with an axis of symmetry (C) about its *C.G.* If n denctes the spin about the axis C and ϕ denotes the Euler's third angle then $A\phi = (A - C)n$.

3) Deduce the principle of energy from the Lagrange's equations.

4) The velocity components of fluid are given by $u = -\frac{c^2 y}{r^2}$, $v = \frac{c^2 x}{r^2}$, $w = 0$, where r is distance from z-axis, find the surfaces which are orthogonal to stream line, the liquid being homogeneous.

5) A thin circular disc of mass M and radius a can turn freely about a thin axis OA, which is perpendicular to its plane and passes through a point O of its circumference. The axis OA is compelled to move in a horizontal plane with angular velocity w about its end A. Show that the inclination θ the vertical of the radius of the disc through O is $\cos^{-1}\left(\frac{g}{aw^2}\right)$ unless $w^2 < \frac{g}{a}$ and then θ is zero.

6) A body moves under no forces about a point O, the principle moments of inertia at O being $6A$, $3A$ and A. Initially the angular velocity of the body has components $w_1 = \text{n}$, $w_2 = 0$, $w_3 = 3n$ about the principal axes. Show that at any later time $w_2 = -\sqrt{5}\,n\ \tanh\sqrt{5}\,nt$ and ultimately the body rotates about the mean axis.

7) In the case of the two-dimensional fluid motion produce by a source of strength m placed at a point S outside a rigid circular disc of radius a whose centre is O. Show that the velocity of slip of the fluid in contact with the disc is greatest at the points where the line joining S to the ends diameter at right angles to OS cut the circle and prove that the magnitude at these points is $\frac{2m.OS}{\left(OS^2 - a^2\right)}$.

8) A mass of fluid moves in such a way that each particle describes a circle in one plane about a fixed axis. Show that the equation of continuity is $\frac{\partial p}{\partial t} + \frac{\partial(\rho w)}{\partial \theta} = 0$, where w be the angular velocity of a particle whose azimuthal angle θ is at time t.

9) Deduce Lagrange's Equations from Hamilton's Principle.

Section - C $\qquad$ 2 × 16 = 32

(Long Answer Type Questions)

Note: Section 'C' contains Four Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to answer in maximum 500 words.

10) A simple circular pendulum is formed of a mass M suspended from a fixed point by a weightless wire of length l; if a mass m, very small compared with M, be knotted on to the wire at a distance 'a' from the point of suspension, show that the time of a small vibration of the pendulum is approximately diminished by $\frac{m}{2M} \cdot \frac{a}{l}\left(1 - \frac{a}{l}\right)$ of itself.

11) A homogeneous sphere of radius a rotating with angular velocity w about a horizontal diameter, is gently placed on a table whose coefficient of friction is μ. Show that there will be slipping at the point of contact for a time $\frac{2aw}{7\mu g}$ and then the sphere will roll with angular velocity $\frac{2w}{7}$.

12) A sphere of radius a is surrounded by infinite liquid of density ρ , the pressure at infinity being π . The sphere is suddenly annihilated. Show that the pressure at a distance r form the centre immediately falls to $\pi\left(1 - \frac{a}{r}\right)$. Show further that if the liquid is brought to rest by impinging on a concentric sphere of radius $\frac{a}{2}$, the impulsive pressure sustained by the surface of this sphere is $\left(7\pi\rho^2 / 6\right)^{1/2}$.

13) A symmetrical top is set in motion on a rough horizontal plane with an angular motion about its axis of figure, the axis being inclined at an angle i to the vertical. Show that between the greatest approach to and recess from the vertical, the centre of gravity describes an arc $h \tan^{-1}\left(\frac{\sin i}{p - \cos i}\right)$ where p and h have their usual meanings.
