MA/ MSCMT-04

December - Examination 2018

M.A. / M.Sc. (Previous) Mathematics Examination

Differential Geometry and Tensors

Paper - MA/ MSCMT-04

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per the given instructions.

Section - A $8 \times 2 = 16$

(Very Short Answer Questions)

Note: Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to 30 words. Each question carries 2 marks.

1) (i) Define an inflexional tangent.

(ii) Define Osculating plane.

(iii) Define Bertrand curves.

(iv) Write down first and second fundamental forms.

(v) Write the statement of Meunier's theorem.

(vi) Write a formula to find principle radii through a point of the surface $z = f(x, y)$

(vii) Define geodesic.

(viii)Define contravariant and covariant vectors.

Section - B $\quad 4 \times 8 = 32$

(Short Answer Questions)

Note: Answer **any four** questions. Each answer should not exceed 200 words. Each question carries 8 marks.

2) Prove that if the circle $lx + my + nz = 0, x^2 + y^2 + z^2 = 2cz$ has three point contact at the origin with the paraboloid $ax^2 + by^2 = 2z$, then $c = (l^2 + m^2)(bl^2 + am^2)$

3) Find the osculating plane at the point t on the helix $x = a\cos t, y = a\sin t, z = ct$.

4) Find the radii of curvature and torsion of a helix. $x = a\cos\theta, y = a\sin\theta, z = a\theta\tan\alpha$.

5) Find the equation of the right conoid generated by lines which meet OZ, are parallel to the plane XOY and intersect the circle $x = a, y^2 + z^2 = r^2$

6) Examine whether the surface $z = y\sin x$ is developable.

7) Prove that the law of transformation of a contravariant vector is transitive.

8) Show that:

   (i) $g^{ij} g^{kl} dg^{ik} = -dg^{jl}$,

   (ii) $g_{ij} g_{kl} dg^{ik} = -dg_{jl}$,

9) If A_{ij} is the curl of a covariant vector, prove that

$$A_{ij,k} + A_{jk,i} + A_{ki,j} = 0$$

Section - C **2 × 16 = 32**

(Long Answer Questions)

Note: Answer **any two** questions. You have to delimit your each answer maximum up to 500 words. Each question carries 16 marks.

10) Find the envelope of the family of planes.

$$F(x,y,z,\theta,\varphi) \equiv \frac{x}{a}\cos\theta\sin\varphi + \frac{y}{b}\sin\theta\sin\varphi + \frac{z}{c}\cos\varphi - 1 = 0$$

11) Find the principal sections and principal curvatures of the surface

$x = a(u+v),\ y = b(u-v),\ z = uv$

12) State and prove Gauss-Bonnet theorem.

13) State and prove the necessary and sufficient condition for a space V_N to be flate.
