MA/MSCMT-03

December - Examination 2018

M.A. / M.Sc. (Previous) Mathematics Examination

Differential Equations, Calculus of Variations and Special Functions

Paper - MA/MSCMT-03

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Type Questions)

Note: Section 'A' contains Very short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Write necessary and sufficient conditions for integrability of differential equation $Pdx + Qdy + Rdz = 0$

(ii) Define Cauchy's problem for second order partial differential equation.

(iii) Write Sturm-Liouville boundary problem in standard form.

(iv) Write necessary condition for extremum of a functional.

(v) What is confluent hypergeometric functions?

(vi) Write Beltrami result for Legendre polynomials.

(vii) Write generating function of Hermite polynomial of order n.

(viii)Write orthogonal property of Laguerre polynomials.

Section - B **4 × 8 = 32**

(Short Answer Type Questions)

Note: Section 'B' contain 08 short Answer Type Questions. Examinees will have to answer any four (4) question. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Solve $y_1 = 2 + \frac{1}{2}\left(x - \frac{1}{x}\right)y - \frac{1}{2}y^2$

3) Solve $pq = x(ps - qr)$

4) Reduce $x\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = x^2, x > 0$ to its canonical form.

5) Find the curve joining the points (x_1, y_1) and (x_2, y_2) that yields a surface of revolution of minimum area when revolved about x-axis.

6) Find the series solution of the linear differential equation

$$4xy'' + 2y' + y = 0$$

7) Derive integral representation of confluent hypergeometric function.

8) Show that $\int_0^\infty e^{-ax} J_0(bx)\, dx = \frac{1}{\sqrt{a^2 + b^2}}$

9) Show that $(n+1)L_{n+1}(x) = (2n+1-x)L_n(x) - nL_{n-1}(x)$.

Section - C $\quad 2 \times 16 = 32$

(Long Answer Type Questions)

Note: Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to answer in maximum 500 words.

10) Solve $\left(y^2z - y^3 + x^2y\right)dx - \left(x^2z + x^3 - xy^2\right)dy + \left(x^2y - xy^2\right)dz = 0$

11) Using the method of separation of variables, solve $\frac{\partial^2 u}{\partial x^2} = 2u + \frac{\partial u}{\partial y}$ given that $u = 0, \frac{\partial u}{\partial x} = 1 + e^{-3y}$ when $x = 0$.

12) Determine the normalized eigenfunctions of the problem

$$y'' + \lambda y = 0 \, ; \, y(0) = 0 \;\; y'(1) + y(1) = 0.$$

13) State and prove orthogonal property of the Lengendre polynomials.

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