MA/MSCMT-02

December - Examination 2018

M.A. / M.Sc. (Previous) Mathematics Examination

Real Analysis and Topology

Paper - MA/MSCMT-02

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Type Questions)

Note: Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Define ring of sets.

(ii) Define Bores measurable function.

(iii) State Weierstrass approximation theorem.

(iv) Write Minkowski's inequality.

(v) Define Hilbert space.

(vi) Define Topological space.

(vii) Define normal space.

(viii) Define compact topological space.

Section - B $4 \times 8 = 32$

(Short Answer Type Questions)

Note: Examinees will have to answer any four (4) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Prove that the outer measure is translation invariants.

3) Show that every bounded measurable function f defined on a measurable set E is L-integrable.

4) Let $< f_n >$ be a sequence of measurable functions defined on a measurable set E and $\lim_{n\to\infty} f_n(x) = f(x)$ on E, then prove that f is measurable on E.

5) Show that a sequence of functions in L^p – space has a unique limit.

6) State and prove Holder's inequality.

7) Prove that homeomorphism is an equivalence relation in the family of topological spaces.

8) Show that regularity is a topological property.

9) Prove that the product space $(X \times Y, W)$ is compact if and only if each of the spaces (X, τ) and (Y, V) is compact.

Section - C **2 × 16 = 32**

(Long Answer Type Questions)

Note: Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to answer in maximum 500 words.

10) Show that every interval is measurable.

11) (i) Show that a function $f: X \to Y$ is continuous iff the inverse image of every closed subset of Y is a closed subset of X.

(ii) Prove that T_∞ is a topology on X_∞.

12) Show that a subset R is connected iff it is an interval.

13) State and prove Riesz-Fisher theorem.