MA/ MSCMT-01

December - Examination 2018

M.A./M.Sc. (Previous) Mathematics Examination

Advanced Algebra

Paper - MA/ MSCMT-01

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per the given instructions.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Questions)

Note: Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to 30 words. Each question carries 2 marks.

1) (i) State Cayley's theorem.

(ii) Define direct product of groups.

(iii) Define composition series.

(iv) Define Kernel of a linear transformation.

(v) Define splitting field.

(vi) Define Galois extension.

(vii) Define nullity of a matrix.

(viii) Define orthonormal set.

Section - B **4 × 8 = 32**

(Short Answer Type Questions)

Note: Answer any four questions. Each question is of 8 marks. Examinee have to delimit each answer in about 200 words.

2) If H and K are subgroups of G with K normal in G. Then show that $H \cap K$ is a normal subgroup of H and $HK / K \cong H/(H \cap K)$

3) Prove that two conjugate classes C[a] and C[b] of a group G are either disjoint or identical.

4) Prove that every finite group G has a composition series.

5) Let $\phi : M \to M'$ be an R - module homomorphism. Then show that Ker ϕ and image set ϕ (M) are sub modules of M and M' respectively.

6) If E is a normal extension of a field F and K is an intermediate field so that F C K C E. Then show that E is also a normal extension of K.

7) Let H be a sub group of all automorphisms of field K. Then show that the fixed field of H is a sub field of K.

8) Let $\{V_1, V_2, ...V_n\}$ be a set of vectors in inner product space V such that they are pairwise orthogonal. Then show that

$$\left\| \sum_{i=1}^{n} V_i \right\|^2 = \sum_{i=1}^{n} \| V_i \|^2$$

9) If $t_1 : V \to V$ and $t_2 : V \to V$ are linear transformation of a finite dimensional inner product space V to it self. Thus show that $(t_1 t_2)^* = t_2^* t_1^*$, where t_1^* denote adjoint of t_1.

Section - C $\qquad 2 \times 16 = 32$

(Long Answer Type Questions)

Note: Answer any two questions. Each question is of 16 marks. Examinee have to delimit each answer in about 500 words.

10) Show that a group G is solvable if and only if $G^{(n)} = \{e\}$ for some $n \in N$.

11) If F is a field, then show that every polynomial $f(x) \in F(x)$ has a splitting field.

12) Let V and V^1 be vector spaces over a field F. Let $\{V_1, V_2, ...V_n\}$ be a basis of V. Then show that there exists a unique linear transformation $t : V \to V^1$ for any list $[b_1^1, b_2^1, ..., b_n^1]$ of vectors in V^1 such that

$t(b_1) = b'_1,\ t(b_2) = b'_2, ..., t(b_n) = b'_n.$

13) Show that every finite dimensional vector space V with an inner product has an orthonormal basis.