MA/MSCMT-04

December - Examination 2017

M.A./M.Sc. (Previous) Mathematics Examination

Differential Geometry and Tensors

Paper - MA/MSCMT-04

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per given instruction. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Type Questions)

Note: Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit is thirty words.

1) (i) Write the equation of oscillating plane.

(ii) Write down the formula of curvature of evolutes.

(iii) Write down the equation of tangent plane to a ruled surface in vector notation.

(iv) Write the parametric equation of anchor ring.

(v) Define Normal section and oblique section of a surface.

(vi) Define zero tensor.

(vii) Explain divergent of a covariant vector.

(viii) Define flat space.

Section - B **$4 \times 8 = 32$**

(Short Answer Type Questions)

Note: Section 'B' contain Eight Short Answer Type Questions. Examinees will have to answer any four (4) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Find the inflexional tangent at (x, y, z) on the surface $y^2 z = 4ax$

3) The necessary and sufficient condition for the curve to be a straight line is that K=0 at all points of the curve.

4) Prove that the torsion of the two Bertrand curves have the same sign and their product is constant.

5) On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the section by the planes z = constant.

6) Prove that osculating plane at any point of a curved asymptotic lines is the tangent plane to the surface.

7) Show that on the surface of a sphere, all great circles are geodesics while no other circle is a geodesic.

8) If A_{ij} is the curl of a covariant vector then prove that

$$A_{ij,k} + A_{jk,i} + A_{ki,j} = 0$$

9) A covariant tensor of first order has components $xy, 2y - z^2, xz$ in rectangular coordinates. Determine its covariant components in spherical coordinate.

Section - C $\quad 2 \times 16 = 32$

(Long Answer Questions)

Note: Section 'C' contain 4 Long Answer Type Questions. Examinees will have to answer any two (2) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) Explain edge of regression. Find the equation of the developable surface whose generating line passes through the curve $y^2 = 4ax, z = 0, x^2 = 4ay, z = c$ and show that its edge of regression is given by $cx^2 - 2ayz = 0 = cy^2 - 3ax(c - z)$.

11) Find the curvature of a normal section of the right helicoids $x = u\cos\phi, y = u\sin\phi, z = c\phi$.

12) Explain geodesic and derive geodesic on surface of revolution given by $x = u\cos\theta, y = u\sin\theta, z = f(u)$.

13) (i) If a Riemannian space $V_N (N>2)$ is isotropic at each point in a region, then prove that the Riemannian curvature is constant throughout that region.

    (ii) If surface of sphere is a two dimensional Riemannian space then Compute the Christoffel Symbols.