MA/MSCMT-09

December - Examination 2017

M.A./M.Sc. (Final) Mathematics Examination

Integral Transforms and Integral Equations

Paper - MA/ MSCMT-09

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Type Questions)

Note: Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) State the relation between Hankel and Laplace transform.

(ii) Define the term "separable kernel".

(iii) State Fredholm's first fundamental theorem.

(iv) State Parseval's theorem for Hankel transform.

(v) Define Integral Equation.

(vi) Define the singular Integral equation.

(vii) Define Inverse Mellin transform.

(viii) Define the convolution of two functions for Fourier transform.

Section - B $4 \times 8 = 32$

(Short Answer Questions)

Note: Section 'B' contain 08 Very Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Find Laplace transform of $8t^2 e^{-1} \sin 4t$.

3) Use partial fractions to find the inverse Laplace Transform of

$$\frac{s^2}{s^4 + 4a^4}$$

4) Find the Fourier cosine transform of e^{-t^2}.

5) State and prove Mellin Inversion theorem.

6) Find the Hankel transform of the function :

$$f(x) = \begin{cases} a^2 - x^2, & 0 < x < a \\ 0, & x > a \end{cases}$$

taking $xJ_0\ (px)$ as the kernel.

7) Show that the function $g(x) = xe^x$ is a solution of the volterra integral equation :

$$g(x) = \sin x + 2\int_0^x \cos(x-t)\, g(t)\, dt$$

8) Using the recurrence relations, find the resolvent kernels of the following kernel -

$$K(x,t) = \sin x \cos t, 0 \le x \le 2\pi, 0 \le t \le 2\pi$$

9) Solve the following integral equation by the method of successive approximations :

$$g(x) = \left(e^x - \frac{1}{2}e + \frac{1}{2}\right) + \frac{1}{2}\int_0^1 g(t)\, dt$$

Section - C $2 \times 16 = 32$

(Long Answer Questions)

Note: Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) An infinite long string having one end $x = 0$ is initially at rest on the x−axis. The end $x = 0$ undergoes a periodic transverse displacement given by $A_0 \sin wt, t > 0$. Find the displacement at any point on the string at any time.

11) Solve : $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} (-\infty < x < \infty, t > 0)$ subject to conditions

$u(x, 0) = f(x)$ and $\left(\frac{\partial u}{\partial t}\right)_{(x,0)} = g(x)$ and $u(x, t)$ is bounded.

12) Solve the Abel integral equation :

(i) $f(x) = \int_0^x \frac{g(t)}{(x-t)^\alpha} dt, 0 < \alpha < 1$

(ii) $\int_0^x \frac{g(t)}{\sqrt{x-t}} dt = 1 + x + x^2$

13) Solve the following symmetric integral equation with the help of Hilbert-schmidt theorem.

$$g(x) = 1 + \lambda \int_0^\pi \cos(x+t) g(t) dt$$