MA/ MSCMT-06

December - Examination 2017

M.A./ M.Sc. (Final) Mathematics Examination

Analysis and Advanced Calculus

Paper - MA/ MSCMT-06

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Type Questions)

Note: Section 'A' contain 8 very short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Define Bounded Linear Transformation for Normed Vector Space.

(ii) Define Second Dual Space.

(iii) State Pythagorean Theorem.

(iv) Define Eigen Values for a Hilbert Space.

(v) Define C^1 - maps.

(vi) Define Locally Lipschitz Function.

(vii) Define Self Adjoint Operator.

(viii)State Polarisation identity in a Hilbert Space.

Section - B **4 × 8 = 32**

(Short Answer Type Questions)

Note: Section 'B' contain 08 short Answer Type Questions. Examinees will have to answer any four (04) question. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) State and prove Minkowaski's Inequality.

3) Let N and N' be normed linear space and D be a subspace of N. Then prove that a linear transformation $T : D \rightarrow N'$ is closed iff its graph T_G is closed.

4) Let M be a closed linear subspace of a Hilbert space H. Let x be a vector not in M and $d = d(x, M)$. Then prove that there exist a unique vector y_0 in M s.t. $\| x - y_0 \| = d$.

5) Show that the set of unitary operators on a Hilbert space H, forms a multiplicative group.

6) State and prove Mean value theorem for Banach space.

7) Let f be a regulated function on a compact interval $[a, b]$ of R into R such that a < b and for all t in $[a, b]$, f (t) ≥ 0. Then prove that $\int_a^b f(t)\,dt \geq 0$. Further prove that if f be a continuous function at a point c of $[a, b]$ and f (c) > 0, then $\int_a^b f(t)\,dt > 0$.

8) Let I be an open interval of R and W be an open subset of a Banach space X over K. Let (t_0, x_0) be a point of I × W and let g be a continuous map of I × W into X. Then prove that a continuous map h: I → W is an integral solution for g at (t_0, x_0) iff for each $t \in$ I,

$$h(t) = x_0 + \int_{t_0}^{t} g(s, h(s))\, ds$$

9) Show that every compact subset of a normed linear space is bounded but its converse need not be true.

Section - C **2 × 16 = 32**

(Long Answer Type Questions)

Note: Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) State and prove Natural embedding theorem for normed linear space.

11) State and prove Risez representation theorem Hilbert space.

12) State and prove Spectral theorem for finite dimensional Hilbert space.

13) State and prove Implicit function theorem on differentiable functions over Banach space.