MA/MSCMT-03

December - Examination 2017

M.A. / M.Sc. (Previous) Mathematics Examination

Differential Equations, Calculus of Variations and Special Functions

Paper - MA/MSCMT-03

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Type Questions)

Note: Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Solve $y^3 \dfrac{d^2y}{dx^2} = c$.

(ii) Write Monge's subsidiary equations.

(iii) Write three dimensional Laplace equation in spherical coordinate system.

(iv) Classify the following partial differential equation:

$$3\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + 5\frac{\partial^2 z}{\partial y} = 0.$$

(v) Define functional.

(vi) Write generating function for Hermite polynomial.

(vii) Write Rodrigues formula for $L_n(x)$.

(viii) Write orthogonal property for Legendre polynomial.

Section - B **$4 \times 8 = 32$**

(Short Answer Type Questions)

Note: Section 'B' contain 08 Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Solve:

$$3x^2\,dx + 3y^2\,dy - (x^3 + y^3 + e^{2z})\,dz = 0$$

3) Solve $r = a^2 t$ by Monge's method.

4) Solve the following Sturm-Liouville problem.

$$y'' + \lambda y = 0;\ \ y'(-\pi) = 0,\ y'(\pi) = 0.$$

5) Find the characteristics of $x^2 r + 2xys + y^2 t = 0$.

6) Find extremals of the functional

$$F[\,y(x)\,] = \int_0^1 \sqrt{1 + y'^2}\,dx, \qquad y(0) = 0,\ y(1) = 2.$$

7) Prove that

$$B(\lambda, c-\lambda)\,{}_2F_1(a,b;c;z) = \int_0^1 t^{\lambda-1}(1-t)^{c-\lambda-1}\,{}_2F_1(a,b;c;zt)\,dt,$$

where $|z| < 1, \lambda > 0, c - \lambda > 0$.

8) Show that

$$(2n+1)(1-x^2)Q_n' = n(n+1)(Q_{n-1} - Q_{n+1})$$

9) Expand x^n in a series of Hermite polynomials.

Section - C **$2 \times 16 = 32$**

(Long Answer Type Questions)

Note: Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words. Use of non-programmable scientific calculator is allowed in this paper.

10) (i) Using the method of separation of variables, solve the two dimensional heat conduction equation.

(ii) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form.

11) State and prove Euler-Lagrange's equation.

12) Solve $x^2 \frac{d^2 y}{dx^2} + (x + x^2)\frac{dy}{dx} + (x - 9)y = 0$ in series.

13) State and prove orthogonal property for Bessel function.