MA/MSCMT-02

December - Examination 2017

M.A. / M.Sc. (Previous) Mathematics Examination

Real Analysis and Topology

Paper - MA/MSCMT-02

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C.

Section - A **8 × 2 = 16**

(Very Short Answer Type Questions)

Note: Section 'A' contain 8 very short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit is thirty words.

1) (i) Define σ ring.

(ii) Define measurable function.

(iii) State Weierstrass approximation theorem.

(iv) State Minkowski's inequality.

(v) Write the necessary and sufficient conditions for a bounded function *f* defined on the interval [a, b], to be L-integrable.

(vi) Define Hilbert space.

(vii) Define exterior of a set

(viii)Define compact topological space.

Section - B **4 × 8 = 32**

(Short Answer Type Questions)

Note: Section 'B' contain 08 short Answer Type Questions. Examinees will have to answer any four (04) question. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Let $\{E_n\}$ be a countable collection of sets of real numbers, then show that.

$$m^*\left(\bigcup_n E_n\right) \leq \sum_n m^*(E_n)$$

3) If f is a bounded measurable function defined on a measurable set E, then prove that $|f|$ is L-integrable over E and

$$\left|\int_E f(x)\,dx\right| \leq \int_E |f(x)|dx$$

4) If a function is summable on E, then show that it is finite almost everywhere on E.

5) Show that an orthonormal system $\{\phi_i\}$ is complete iff it is closed.

6) State and prove Holder's inequality.

7) Prove that in a T_2– space, a convergent sequence has a unique limit.

8) Show that the property of a space being a Haudorff space is a hereditary property.

9) Prove that every open continuous image of a locally compact space is locally compact.

Section - C **2 × 16 = 32**

(Long Answer Type Questions)

Note: Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) (i) Show that every bounded measurable function f defined on a measurable set E is L-integrable.

(ii) If the function f and g are Lebesgue integrable over the measurable set E and if $f(x) < g(x)$ on E, then prove that

$$\int_E f(x)\,dx \leq \int_E g(x)\,dx$$

11) (i) Is $\overline{A \cap B} = \overline{A} \cap \overline{B}$? Give reason in support of your answer.

(ii) Show that every metric space is a T_2-space.

12) Prove that every interval is measurable.

13) State and prove Riesz-Fisher theorem.

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