MA/MSCMT-01

December - Examination 2017

M.A./M.Sc. (Previous) Mathematics Examination

Advanced Algebra

Paper - MA/MSCMT-01

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Type Questions)

Note: Section 'A' contains 08 Very Short Answer type questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Define internal direct product of groups.

(ii) Write class equation of a group.

(iii) Define solvable sub group.

(iv) Define prime element in a ring.

(v) Define module endomorphism.

(vi) Define simple field extension.

(vii) Define Galois extension of a field.

(viii) Define nullity of a matrix.

Section - B **4 × 8 = 32**

(Short Answer Type Questions)

Note: Section 'B' contains 08 Short Answer type questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Prove that the set of all automorphism of G Aut (G) is a group under composition of automorphisms.

3) Prove that a group G is abelian if and only if $G' = \{e\}$; where e is identity of G and G' is derived subgroup of G.

4) If a group G has a solvable homomorphic image whose kernel is solvable, them prove that the group is solvable.

5) Let F be a field and $p(x) \in F[x]$ such that deg $p(x) = n,\ n \geq 1$. Then prove that there exist a finite extension K of F in which $p(x)$ gets a full set of n roots such that $[K : F] \leq n!$

6) For every prime p and $n \geq 1$, prove that these exists a finite field with p^n elements.

7) Prove that every orthonormal set of vectors is a linearly independent set in an inner product space.

8) Let V be a finite dimensional inner product space and W be its any subspace. Then prove that V is the direct sum of W and $W^{\perp}$.

9) Prove that a polynomial of degree n over a field F can have at most n roots in any extension field.

Section - C $\quad 2 \times 16 = 32$

(Long Answer Type Questions)

Note: Section 'C' contain 04 Long Answer type questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) State and prove unique factorizaton theorem.

11) Let M be an R-module and N_1 , N_2 , ... , Nk be sub module of m. Then prove that the following statements are equivalent:

    (i) $M = N_1 \oplus N_2 \oplus \ldots\ldots\ldots\ldots \oplus N_k$

    (ii) $pf = n_1 + n_2 + \ldots\ldots\ldots + n_k = 0$ then

    $n_1 = n_2 = \ldots\ldots\ldots\ldots = n_k = 0$ for $n_i \in N_i$

    and

    (iii) $N_i \cap (N_1 + N_2 + \ldots\ldots + N_{i-1} + N_{i+1} + \ldots\ldots + N_k) = \{0\}$

12) Prove that any two finite dimensional vector space over the same field are isomorphic if and only if they are of same dimension.

13) Let V and V' be finite dimensional vector space over a field F with bases B and B' respectively. If $t : v = v'$ if a linear transformation then prove that

    $$M_{B^*}^{B'^*}(t^*) = \left[M_{B'}^{B}(t)\right]^T$$

    Where t^* is the dual map of t and B^*, B'^* are the dual bases of B and B' respectively.
