MA/MSCMT-10

December - Examination 2016

M.A./M.Sc. (Final) Mathematics Examination

Mathematical Programming

Paper - MA/MSCMT-10

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answer as per the given instructions.

Section - A $8 \times 2 = 16$

Very Short Answer Questions

Note: Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Explain supporting Hyperplane.

(ii) Define bounded variable problem.

(iii) Explain importance of integer programming problem.

(iv) Write Kuhn-Tucker conditions for min $f(x)$ and $x = (x_1, x_2 \ldots\ldots\ldots x_n)$ $g_j(x) \geq 0$ $(j = 1, 2 \ldots\ldots m)$

(v) Define general quadratic programming problem.

(vi) Write the dual of quadratic programming problem.

$$\max f(x) = C^T X + \frac{1}{2} X^T G X$$

subject to AX = b, X ≥ 0

(vii) Define convex separable programming problem.

(viii) Explain Bellman's principle of optimality.

Section - B **4 × 8 = 32**

Short Answer Questions

Note: Section 'B' contain 08 Short Answer Type Questions. Examinees will have to answer any four (04 questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Prove that the sum of convex function is convex and if atleast one of the function is strictly convex then prove that the sum is strictly convex.

3) Solve the following L.P.P. with the help of revised simplex method but without use of artificial variable.

$$\max .z = 2x_1 - 6x_2$$

$$x_1 - 3x_2 \leq 6$$

$$2x_1 + 4x_2 \geq 8$$

$$-x_1 + 3x_2 \leq 6$$

$$x_1 ,\ x_2 \geq 0$$

4) Use branch and bound method to solve the following I.P.P.

$$\min .z = 4x_1 + 3x_2$$

$$s.t.\ 5x_1 + 3x_2 \geq 30$$

$$x_1 \leq 4$$

$$x_2 \leq 6$$

$x_1 ,\ x_2 \geq 0$ and are integers.

5) Obtain necessary and sufficient conditions for optimum solution of the following Non Linear programming problem.

$$\min. z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$$

s.t. $x_1 + x_2 + x_3 = 15$

$2x_1 + x_2 + 2x_3 = 30$

$x_1, x_2, x_3 \geq 0$

6) Solve the following non-linear programming problem using the method of Lagrange's multipliers.

$$\min f(x) = x_1^2 + x_2^2 + x_3^2$$

s.t. $4x_1 + x_2^2 + 2x_3 = 14$

$x_1, x_2, x_3 \geq 0$

7) Derive the dual of quadratic programming problem

$$\min f(x) = C^T X + \frac{1}{2} X^T G X$$

s.t. $AX \geq b$

where A is an mxn real matrix and G is an nxn real positive semi definite a symmetric matrix.

8) Solve the following L.P.P. by using dynamic programming.

$$\max . z = 3x_1 + 5x_2$$

s.t. $x_1 \leq 4$

$x_2 \leq 6$

$3x_1 + 2x_2 \leq 18$

$x_1, x_2 \geq 0$

9) Use dynamic programming to solve:

$$\min. \left(x_1^2 + x_2^2 + x_3^2\right)$$

s.t. $x_1 . x_2 . x_3 \geq 50$

$x_1, x_2, x_3 \geq 0$

Section - C $\quad 2 \times 16 = 32$

Long Answer Questions

Note: Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words. Use of non-programmable scientific calculator is allowed in this paper.

10) Solve the integer programming problems.

$$\max z = 2x_1 + 10x_2 - 10x_3$$
$$2x_1 + 20x_2 + 4x_3 \leq 15$$
$$6x_1 + 20x_2 + 4x_3 = 20$$
$$x_1, x_2, x_3 \geq 0 \text{ and are integers.}$$

11) Use Kuhn-Tucker conditions to solve.

$$\text{Optimize } f(x) = 2x_1 + 3x_2 - \left(x_1^2 + x_2^2 + x_3^2\right)$$
$$\text{s.t.} \quad x_1 + x_2 \leq 1$$
$$2x_1 + 3x_2 \leq 6$$
$$x_1 , \; x_2 \geq 0$$

12) Solve the quadratic programming problem by Beale's method.

$$\max. f(x_1 x_2) = x_1 + x_2 - x_1^2 + x_1 x_2 - 2x_2^2$$
$$\text{s.t.} \quad x_1 , \; x_2 \geq 0$$

13) Find optimal solution of the convex separate programming problem.

$$\max z = 3x_1 + 2x_2$$
$$\text{s.t.} \quad 4x_1^2 + x_2^2 \leq 16$$
$$x_1 , \; x_2 \geq 0$$