MA/MSCMT-09

December - Examination 2016

M.A./M.Sc. (Final) Mathematics Examination

Integral Transforms and Integral Equations

Paper - MA/MSCMT-09

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answer as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section - A $8 \times 2 = 16$

Very Short Answer Questions

Note: Answer **all** questions. As per the nature of the question should be given in 30 words. Each question carries 02 marks.

1) (i) Write definition of Laplace transform.

(ii) Write definition of Fourier transform.

(iii) Write definition of Mellin transform.

(iv) Write relation between Hankel and Laplace transform.

(v) Write definition of linear integral equation.

(vi) Define degerate kernel.

(vii) Define eigen values.

(viii) Define orthogonal functions.

Section - B **4 × 8 = 32**

Short Answer Questions

Note: Answer **any four** questions. Each answer should be given in 200 words. Each question carries 8 marks.

2) Evaluate $L^{-1}\left[\frac{1}{(p-4)^5}+\frac{5}{(p-2)^2+25}+\frac{p+3}{(p+3)^2+36}\right]$

3) Apply convolution theorem to prove that

$$\beta(m,n)=\int_0^1 u^{m-1}(1-u)^{n-1}\,du=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)},m>0,\ n>0.$$

4) Find the Fourier cosine transform of e^{-t^2}.

5) If F(p) and G(p) are the Mellin transforms of the functions $f(x)$ and $g(x)$. Then show that

$$M\{f(x)g(x);p\}=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}F(z)G(p-z)\,dz.$$

6) Find the Hankel transform of the function

$$f(x)=\begin{Bmatrix} a^2-x^2 & 0<x<a \\ 0, & x>a \end{Bmatrix}$$

7) Form an integral equation corresponding to the differential equation.

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

with initial conditions $y(0) = 1, y'(0) = 0.$

8) Solve the homogeneous Fredholm integral equation of the second kind.

$$g(x) = \lambda \int_0^{2\pi} \sin(x + t) g(t) dt.$$

9) If $y(x)$ is continuous and satisfies the integral equation

$$y(x) = \lambda \int_0^1 K(x,t)\, y(t)\, dt,$$

where $K(x, t) = \begin{cases} (1 - t)x, & 0 \le x \le t \\ (1 - x)t, & t \le x \le 1 \end{cases}$

Then prove that $y(x)$ is also the solution of the boundary value problem $\frac{d^2y}{dx^2} + \lambda y = 0;\ y(0) = 0,\ y(1) = 0.$

Section - C $\qquad 2 \times 16 = 32$

Long Answer Questions

Note: Answer **any two** questions. Answer of each question should be given in 500 words. Each question carries 16 marks.

10) Solve the following differential equation using Laplace transform

$$t\,y'' + (t-1)y' - y = 0;\ \ y(0) = 5,\ \ y(\infty) = 0.$$

11) Find the Fourier transform of $f(t) = \begin{cases} 1, |t| < a \\ 0, |t| > a \end{cases}$

and hence prove that $\int_0^\infty \frac{\sin^2 at}{t^2} dt = \frac{\pi a}{2}$.

12) Show that the solution of Laplace equation for U inside the semi-infinite strip $x > 0.\ 0 < y < b$, such that

$$U = f(x),\ \ when\ y = 0, 0 < x < \infty$$
$$U = 0,\ \ when\ y = b, 0 < x < \infty$$
$$U = 0,\ \ when\ x = 0, 0 < y < b$$

is given by $U = \frac{2}{\pi} \int_0^\infty f(u)du \int_0^\infty \frac{\sin h(b-y)p}{\sin h\ pb} \sin xp \sin up\, dp$

13) Solve the integral equation

$$g(x) = x + \lambda \int_{-\pi}^{\pi} (x \cos t + t^2 \sin x + \cos x \sin t)\, g(t)\, dt.$$
