MA/ MSCMT-05

December - Examination 2016

M.A. / MSc. (Previous) Mathematics

Examination

Mechanics

Paper - MA/ MSCMT-05

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **8 × 2 = 16**

(Very Short Answer Questions)

Note: Section 'A' contain (08) Very Short Answer Type Questions. Examinees have to attempt **all** questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Write definition of moment of inertia.

(ii) Define simple equivalent pendulum.

(iii) Define centre of percussion.

(iv) Define two dimensional motion under finite forces.

(v) Define instantaneous axis of rotation.

(vi) Write down the equations of motion for impulsive forces.

(vii) State the principle of conservation of angular momentum under finite forces.

(viii) What is the distinction between Hamilton's principle and Principle of least action?

Section - B

4 × 8 = 32

(Short Answer Questions)

Note: Section 'B' contain 08 Short Answer Type Questions. Examinees have to delimit each answer in maximum 200 words.

2) State and prove D' Alembert's principle.

3) Derive Euler's geometrical equations of motion.

4) A body moves under no forces about a point O, the principal moments of inertia at 0 being 6A, 3A and A. Initially the angular velocity of the body has components $w_1 = n, w_2 = 0, w_3 = 3n$ about the principal axes. Show that at any later time $w_2 = -\sqrt{5}\, n \tan h\sqrt{5}\, nt$ and ultimately the body rotates about the mean axis.

5) A circular plate is turning in its own plane about a point A on its circumference. Suddenly A is freed and point B, also on the circumference, fixed show that the plate will be reduced to rest if the arc AB is one third of the circumference.

6) A circular disc, of radius a, has a thin rod pushed through its centre perpendicular to its plane, the length of the rod being equal to the radius of the disc. Show that the system cannot spin with the rod vertical unless the angular velocity is greater then $\sqrt{\frac{20g}{a}}$.

7) The velocity components for a two dimensional flow system can be given in the Eulerian system by

$u = 2x + 2y + 3t; v = x + y + \frac{t}{2}$ Find the displacement of a fluid particle in the Lagrangian system.

8) Show that

$u = \frac{2xyz}{(x^2 + y^2)^2}$, $v = \frac{(x^2 - y^2)_z}{(x^2 + y^2)^2}$ and $w = \frac{y}{(x^2 + y^2)}$

are the velocity components of a possible fluid motion. Is this motion irrotational?

9) State and prove Bernoulli's theorem.

Section - C

2 × 16 = 32

(Long Answer Questions)

Note: Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) A solid homogeneous cone of height h and vertical angle α, oscillate about a horizontal axis through its centre. Show that the length of the simple equivalent pendulum is

$$\frac{h}{5}(4+\tan^2\alpha)$$

11) Deduce the principle of energy from the Lagrange's equation.

12) Prove that the Cauchy's integrals are the integral of the Helmoholtz equations.

13) Show that the velocity potential.

$$\phi = \frac{1}{2}\log\frac{(x+a)^2+y^2}{(x-a)^2+y^2}$$

gives a possible motion. Determine the stream lines and show also that the curves of equal speed (q = constant) are ovals of Cassinni given by $r\,r'$ = constant.