

MA/MSCMT-04

December - Examination 2016

M.A. / M.Sc. (Previous) Mathematics Examination**Differential Geometry and Tensors****Paper - MA/MSCMT-04****Time : 3 Hours]****[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **$8 \times 2 = 16$**

Note: Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be 30 words.

- 1) (i) Define skew-curvature.
- (ii) Define osculating plane.
- (iii) Write formula for Torsion of the involute.
- (iv) Interpretate envelope of family of curves geometrically.
- (v) Define surface of revolution.
- (vi) Write geometrically significance of second fundamental theorem.
- (vii) Define trajectory of family of curves.
- (viii) Define skew symmetric tensor.

Section - B**4 × 8 = 32**

Note: Section 'B' contain 08 Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Prove that divergence of Einstein tensor vanishes.

3) If A_{ij} is the curl of a covariant vector then prove that

$$A_{ij,k} + A_{jk,i} + A_{ki,j} = 0$$

4) Prove that outer multiplication of tensors is commutative and associative.

5) Find the lines that have four point contact at $(0, 0, 1)$ with the surface

$$x^4 + 3xyz + x^2 - y^2 - z^2 + 2yz - 3xy - 2y + 2z - 1 = 0$$

6) Prove that the principal normals at consecutive points of a curve do not intersect unless $\tau = 0$

7) Find the evolutes of circular helix

$$x = a \cos \theta, \quad y = a \sin \theta, \quad z = a \theta \tan \alpha$$

8) Prove that the metric of a surface is invariant under parametric transformation.

9) State and prove Bonnet's theorem for parallel surface.

Section - C **$2 \times 16 = 32$**

Note: Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

- 10) (i) Prove that Christoffel symbols of first kind are not tensor quantities.
- (ii) Prove that Riemannian co-ordinates are geodesic co-ordinates with the pole at P_0 .
- 11) (i) Prove that the generators of a developable surface are tangents to curve.
- (ii) On the paraboloid $x^2 - y^2 = z$ find the orthogonal trajectories of sections by the planes $z = \text{constant}$.
- 12) (i) Show that the surface $e^z \cos x = \cos y$ is minimal surface.
- (ii) Show that conjugate direction at a point P on a surface are parallel to conjugate diameters of the indicatrin at point P.
- 13) Explain geodesic on a surface of revolution and derive expressions for complete integral of the differential equation of geodesic on the surface of revolution.
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