MA/MSCMT-03

December - Examination 2016

M.A. / M.Sc. (Previous) Mathematics

Examination

Differential Equations, Calculus of Variations and Special Functions

Paper - MA/MSCMT-03

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Questions)

Note: Section 'A' contain 08 very short answer type questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Write condition of integrability for a total differential Equation.

(ii) Write General Riccati's equation

(iii) Write two dimensional Laplace equation in Polar Coordinate system.

(iv) Classify the following PDE as hyperbolic, parabolic or elliptic $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$

(v) Write Integral form of Gauss Hypergeometric function.

(vi) Write Bessel's function of first kind of Index n.

(vii) Write generating function of Hermite Polynomial.

(viii) Which is the extremizing curve of the brachistochrone problem? a Circle, a Catenery, a Cycloid or a Straight Line?

Section - B **4 × 8 = 32**

(Short Answer Questions)

Note: Section 'B' contain 08 short answer type questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Solve the following differential equation.

$$2\sin x \frac{d^2y}{dx^2} + 2\cos x \frac{dy}{dx} + 2\sin x \frac{dy}{dx} + 2y\cos x = \cos x$$

3) Find the shape of the curve on which a bead is sliding from rest and accelerated by gravity will ship (without friction) in least time from one point to another.

4) Show that a surface passing through the circle $z = x^2 + y^2 = 1$ and satisfying the differential equation s $= 8xy$ is $z = (x^2 + y^2)^2 - 1$

5) Use the method of separation of variable to solve the equation

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \text{ given that } u(x,0) = 6e^{-3x}$$

6) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to Canonical form and solve it.

7) If $|z| < 1$ and if Re $(c) >$ Re $(b) > 0$; then Prove that

$$B(b, c-b)_2 F_1(a,b;c;z) = \int t^{b-1}(1-t)^{c-b-1}(z = zt)^{-a}\, dt$$

8) Discuss the relation between $J_n(x)$ *and* $J_{-n}(x), n$ being and integer.

9) Prove the following recurrence relation for Leguerre's polynomial

$$(n+1)L_{n+1}(x) = (2n+1-x)L_n(x) - nL_{n-1}(x)$$

Section - C **2 × 16 = 32**

(Long Answer Questions)

Note: Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words. Use of non-programmable scientific calculator is allowed in this paper.

10) Solve

(i) $\left(\frac{d^3y}{dx^3}\right)^2 + x\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} = 0$

(ii) $2r + (p+x)s + yt + y(rt - s^2) + q = 0$

11) (i) Prove that the necessary and sufficient condition for total differential equation $Pdx + Qdy + Rdz = 0$ to be integrable is

$$P\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) - Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) = 0$$

(ii) Solve in series $2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1 - x^2)y = 0$

12) (i) Find the eigenvalues and eigen functions for the boundary value problem $y^n + \lambda y = 0$ under the boundary condition $y(a) = 0$ and $y(b) = 0$, $0 < a < b$; a, b are arbitrary real constants.

(ii) Show that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$

13) (i) Prove that

$$\int_0^a x(a^2 - x^2)J_0(kx)dx = \frac{4a}{k^3} J_1(ak) - \frac{2a^2}{k^2} J_0(ak)$$

(ii) Prove $\mathrm{H}_n(x) = (2x)^n \, {}_2F_0\left(-\frac{n}{2}, \frac{1-n}{2}; -; -\frac{1}{x^2}\right)$