MA/MSCMT - 02

December - Examination 2016

M.A./M.Sc. (Previous) Mathematics Examination

Real Analysis and Topology

Paper - MA/MSCMT - 02

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **8 × 2 = 16**

(Very Short Answer Questions)

Note: Section 'A' contain (08) Very Short Answer Type Questions. Examinees have to attempt **all** questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Define σ ring.

(ii) Define measurable set.

(iii) State Weierstrass's theorem.

(iv) State Fatou's lemma for measurable function.

(v) Define Hilbert space.

(vi) State Parseval's identity.

(vii) Define normal space.

(viii) Define directed set.

Section - B $4 \times 8 = 32$

(Short Answer Questions)

Note: Section 'B' contain 08 Short Answer Type Questions. Examinees have to delimit each answer in maximum 200 words.

2) Let \$ be an algebra of sets of a set X and $\{A_n\}$ be a sequence of sets in \$. Then show that there exists a sequence $\{B_n\}$ of sets in \$ such that $B_i \cap Bj = \phi$ if $i \neq j$ and $\cup_{i=1}^{\infty} B_i = \cup_{i=1}^{\infty} A_i$.

3) Let E be a measurable set. Then for any real number x, prove that the translation $E + x$ is also measurable. Further, prove that.

4) Prove that every bounded measurable function f defined on a measurable set E is L-integrable over E.

5) Prove that the space L_2 of square summable functions is linear space.

6) Is $\overline{A \cap B} = \overline{A} \cap \overline{B}$? Give reason in support of your answer.

7) Prove that a one one-one onto continuous map $f:(X, \tau) \to (Y, \mu)$ is a homeomorphism if f is either open or closed.

8) Show that T_∞ space is a topology on X_∞.

9) Prove that the product space $X \times Y$ is connected if and only if X and Y are connected.

Section - C **2 × 16 = 32**

(Long Answer Questions)

Note: Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions Each questions is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) (i) Prove that continuous image of a connected space is connected.

(ii) State and prove Alexander subset lemma for compact topological space.

11) (i) Prove that every T_3 - space is a T_2 - space.

(ii) State and prove Minkowski's inequality.

12) Prove that the necessary and sufficient condition for a bounded function f defined of interval $[a, b]$ to be L-integrable over $[a, b]$ is that for given $\in > 0$, there exists a measurable partition P of $[a, b]$ such that $U(f, p) - L(f, p) < \in$.

13) State and prove Riesz-Fisher theorem.