MCA - 09

December - Examination 2015

MCA IInd Year Examination

Discrete Mathematics

Paper - MCA - 09

Time : 3 Hours] **[Max. Marks :- 80**

Note : The question paper is divided into three sections A, B and C. Use of calculator is allowed in this paper.

Section - A 8 x 2 = 16

Note : Section 'A' contain 08 very short answer type questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit is 30 words.

1) (i) Express the following set in Roster method:

$A = \{x : x \text{ is a day of the week}\}$

(ii) If A = {1, 2}, Write down all the bijective functions from A to itself.

(iii) Write the negation of the following statement: p:7 is a rational.

(iv) Define Boolean Algebra.

(v) Define quantifier.

(vi) State the Pigeonhole principle.

(vii) Define identity element for a binary operation.

(viii) Define a normal subgroup of a group.

Section - B 4 x 8 = 32

Note : Section 'B' contain 08 short answer type questions. Examinees will have to answer any four 04 questions. Each question is of 08 marks Examinees have to delimit each answer in maximum 200 words.

2) Let $R^2 = \{(a, b) : a, b \text{ are reals}\}$
We define
$(a, b) + (c, d) = (a + c, b + d)$,
$(a, b) \cdot (c, d) = (ac, bd)$
a, b, c, d are reals.
Show that R^2 is a commutative ring with unity.

3) Show that the simple graphs with the following adjacency matrices are isomorphic

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

4) Prove that a group of order less than or equal to 5 is Abelian.

5) How many different words can be formed with the letters of the word 'COMPUTER'? How many of them

(i) will begin with C and with R?

(ii) will not have T and R together?

6) Let P(S) be the set of all subsets of a non-empty set S i.e. power set of sets. Let the operations sum (+), product (.) and complement (‘) be defined on P(S) as follows:

$A + B = A \cup B$

$A . B = A \cap B$

and $A' = S - A$ for all $A, B \in P(S)$

Show that (P (S), +,.,‘) is a Boolean algebra.

7) Verify the validity of the following argument.

All men are mortal Socrates is a man.

Therefore Socrates is a mortal.

8) Prove that,

$(\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R.$

9) Let N be the set of natural numbers.

Let $f : N \rightarrow N, f(x) = x^2 + 1$

Examine f for (i) one - one (ii)onto.

Section - C 2 x 16 = 32

Note : Section ‘C’ contain 04 Long answer type questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) (i) Let N be the set of natural numbers. Let a relation R be defined on N × N by (a, b) R (c, d) if and only if $ab = bc$. Show that R is an equivalence relation on N × N.

(ii) In a class of 80 students, everybody can speak either English or Hindi or both. If 39 can speak English, 62 can speak Hindi, how many can speak both the languages?

11) (i) Prove that any two left (or right) cosets of a subgroup are either disjoint or identical.

(ii) Prove that the intersection of two subgroups of a group is again a subgroup of the group.

12) Draw the following graphs:

(i) 3-regular but not complete

(ii) 2-regular but not complete

(iii) A complete bipartite graph having 2 vertices in one partite set and 4 vertices in the other partite set.

13) (i) In how many ways can 5 men and 4 women dine at a round table if no two women are to sit together.

(ii) Show that $(A - B) - C = (A - C) - (B - C)$