MA/MSCMT - 10

December - Examination 2015

M.A./M.Sc. Final Mathematics Examination

Mathematical Programming

Paper - MA/MSCMT - 10

Time : 3 Hours] **[Max. Marks :- 80**

Note : The question paper is divided into three Sections A, B, and C. Use of calculator is allowed in this paper.

Section - A 8 x 2 = 16

Note : Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit is thirty words.

1) (i) Define a convex function.

(ii) Define a Integer programming problem (I.P.P.).

(iii) Write the objectives function

$$Z = x_1^2 + 2x_1x_2 + 46x_1x_3 + 3x_2^2 + 2x_2x_3 + 5x_3^2 + 4x_1 - 2x_2 + 3x_3$$

In the form $Z = X^T.AX + q^T X$

(iv) Define a relation between saddle point of $F(X, \lambda)$ and minimal point of $F(X)$.

(v) Write the Lagrange's function in the following nonlinear programming problem.

$$\text{Min.} f(x_1, x_2) = x_1^2 - 2x_1 - x_2$$
$$\text{s.t. } 2x_1 + 3x_2 \leq 6$$
$$2x_1 + x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

(vi) Write down the quadrate from whose associated matrix is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(vii) Explain Bellman's principal of optimality.

(viii)Define a separable programming problem.

Section - B

4 x 8 = 32

Note : Section 'B' contain 08 Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Solve the following linear programming problem using revised simplex method.

$$Min.z = x_1 + x_2$$
$$s.t. x_1 + 2x_2 \geq 7$$
$$4x_1 + x_2 \geq 6$$
$$x_1 \geq 0, x_2 \geq 0$$

3) Solve the following I.P.P.

$Min.z = 9x_1 + 10x_2$
$s.t. 4x_1 + 3x_2 \geq 40$
$x_1 \leq 9$
$x_2 \leq 8$
$x_1 \geq 0, x_2 \geq 0$
and x_1, x_2 are integers.

4) Divide a number 'a' into 3 parts such that continuous product of first, square of second, cube of third is maximum using Lagrange's multiplier method.

5) If $f(X)$ is a concave function, then find the dual to the following quadratic programming problem:

$Min.f(X) = C^T X + \frac{1}{2} X^T GX$
$s.t. AX \leq b$
$X \geq 0$

6) Use dynamic programming to solve

$Min.f(x) = x_1^2 + x_2^2 + x_3^2$
$s.t. x_1, x_2, x_3 = 10$
$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

7) Use dynamic programming to solve following L.P.P.

$$Max.z = 3x_1 + 7x_2$$
$$s.t. x_1 + 4x_2 \le 8$$
$$x_2 \le 8, \text{ and}$$
$$x_1 \ge 0, x_2 \ge 0$$

8) Determine the sign of definiteness for each of the following matrices:

(a) $\begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 2 & 2 & 10 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & 5 \\ 3 & 2 & -3 \end{bmatrix}$

9) Show that $f(x_1, x_2) = x_1 x_2$ is not a convex set over E^2.

Section - C 2 x 16 = 32

Note : Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) Apply Wolfe's method to solve

$$Max. f(X) = 6x_1 + 3x_2 - 4x_1x_2 - 2x_1^2 - 3x_2^2$$
$$s.t. x_1 + x_2 \le 1$$
$$2x_1 + 3x_2 \le 4$$
$$x_1 \ge 0, x_2 \ge 0$$

11) Solve the following convex separable programming problem

$$Max.z = 2x_1 - x_1^2 + x_2$$
$$s.t.2x_1 + x_2 \leq 4$$
$$2x_1 + 3x_2 \leq 6$$
$$x_1 \geq 0, x_2 \geq 0$$

12) Solve the following non linear programming problem using Kuhn-Tucker conditions.

$$Max.z = 10x_1 + 10_{x_2} - x_1^2 - x_2^2$$
$$s.t.x_1 + x_2 \leq 14$$
$$-x_1 + x_2 \leq 6$$
$$x_1 \geq 0, x_2 \geq 0$$

13) Solve the following bounded variable problem:

$$Max.z = 4x_1 + 2x_2 + 6x_3$$
$$s.t.4x_1 - x_2 \leq 9$$
$$-x_1 + x_2 + 2x_3 \leq 8$$
$$-3x_1 + x_2 + 4x_3 \leq 12$$
$$1 \leq x_1 \leq 3, 0 \leq x_2 \leq 5, 0 \leq x_3 \leq 2$$