# MA / MSCMT - 06

## December - Examination 2015

## M.A./ M.Sc. (Final) Examination

## Analysis and Advanced Calculus

## Paper - MA / MSCMT - 06

**Time : 3 Hours ]** **[ Max. Marks :- 80**

**Note :** The question paper is divided into three sections A, B and C. Use of calculator is allowed in this paper.

**Section - A** 8 x 2 = 16

**Note :** Section 'A' contain 08 very short answer type questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit is thirty words.

1) (i) Define n-Dimensional unitary space.

(ii) Define functional and conjugate space.

(iii) Define product space of normal linear space.

(iv) State the uniform boundedness theorem.

(v) Define directional derivative.

(vi) State Reisz Lemma.

(vii) Define the derivative of a map.

(viii) Define Hilbert space.

**Section - B** 4 x 8 = 32

**Note :** Section 'B' contain 8 short answer type questions. Examinees will have to answer any four 04 questions. Each question is of 08 marks Examinees have to delimit each answer in maximum 200 words.

2) Show that every convergent sequence in a normal linear space is a Cauchy sequence.

3) If T be a linear transformation of a normed linear space N into normed linear space N′. Then prove that inverse of T i.e. $T^{-1}$ exists and is continuous on its domain of definition if ∃ *a* constant $K \geq 0$ s.t. $K\,||x|| \leq ||T(x)||\ V\, x \in N$.

4) If B and B′ be Banach spaces and T is a continuous linear transformation from B on B′, then prove that T is an open mapping.

5) If A be the set of all self-adjoint operator in β(H), then show that A is a closed linear subspace of β(H) and therefore A is a real Banach space containing the identity transformation.

6) If P is a projection on a Hilbert space H, then prove that :

(i) $||P_x|| \leq ||x|| \quad \forall\, x \in H$

(ii) $||P|| \leq I$

(iii) P is a positive operator.

(iv) $0 \leq P \leq I$

7) If X be banach space over the field K of scalars and V be an open subset of X. If $f : V \rightarrow R$ be a function. Let u and v be any two distinct points in V s.t. $[u, v] \subset V$ and $f$ is differentiable at all points of $[u, v]$ then show that:

$f(v) - f(u) = Df(u + t(v - u))\,.\,(v - u)$

where $t \in (0, 1)$

8) Let X be a Banach space over the field k os scalars, and let I be an open interval in R containing [0 , 1]. If $\psi : I \to X$ is $(n + 1)$ times continuously differentiable function of single variable $t \in I$ then show that:

$$\Psi(1) = \Psi(0) + \Psi'(0) + \frac{\Psi''(0)}{2!} \dots + \frac{\Psi^{n}(0)}{n!} + \int_0^1 \frac{(1-t)^n}{n!} \Psi^{n+1}(t)\,dt$$

9) If f be a continuous function on a compact interval $[a, b]$ of R into a Banach space X over K. Let F be the function $t \to \int_0^1 f$ on $[a, b]$ into X. Let $g$ be any differentiable function on $[a, b]$ into X such that $Dg = f$, then prove that F is differentiable, DF = $f$ and

$$\int_a^b f + F(b) - F(a) = g(b) - g(a)$$

**Section - C** 2 x 16 = 32

**Note :** Section 'C' contain 04 Long answer type questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) (i) State the prove Minkowski's inequality.

(ii) If N be a normed linear space, then show that the two norms $\| \|_1$ and $\| \|_2$ defined in N are equivalent iff positive real numbers $a$ and $b$ s.t.

$$a \| x \|_1 \leq \| x \|_2 \leq b \| x \|_1, \forall x \in N$$

11) If $\{e_i\}$ be an orthonormal set in Hilbert space H and $x$ be an arbitrary vector in H, show that

$$x - \sum (x, e_i)\, e_i \perp e_i\ f \text{ or } \forall j$$

12) (i) State and prove that the Mean value theorem.

(ii) State and prove the inverse function theorem.

13) If f be a functional defined on a linear subspace M of a complex normed linear space $N$, $x_0 \in M$ and

$$M_0 = [M \cup \{x_0\}] = \{x + \alpha\, x_0 : x \in M \text{ and } \alpha \text{ is real}\}$$

is the linear subspace spanned by M and $x_0$, then f can be extended to a functional $f_0$ defined on $M_0$ s.t. $\|f_0\| = \|f\|$.

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