MA / MSCMT - 04

December - Examination 2015

M.A. / M.Sc. Previous Examination

Differential Geometry and Tensors

Paper - MA / MSCMT - 04

Time : 3 Hours] **[Max. Marks :- 80**

Note : The question paper is divided into three sections A, B and C. Use of calculator is allowed in this paper.

Section - A 8 x 2 = 16

Note : Section 'A' contain 08 very short answer type questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit is 30 words.

1) (i) Define Isotropic point.

(ii) Write Buler's differential equation for functional $f(x^i, x^p)$.

(iii) Define gradiant of a scalar.

(iv) Write the Weingarten formulae.

(v) Define the principal curvature.

(vi) Define normal section and oblique section of a surface.

(vii) Write down the equation of tangent plane to a ruled surface in vector notation.

(viii) Define the oscillating circle.

Section - B 4 x 8 = 32

Note : Section 'B' contain 8 short answer type questions. Examinees will have to answer any four 04 questions. Each question is of 08 marks Examinees have to delimit each answer in maximum 200 words.

2) Show that the tangent at any point of the curve whose equation are $x = 3t, y = 3t^2, z = 2t^3$ makes a constant angle with line $y = z - x = 0$.

3) If the tangent and binormal at a point of curve make angles θ, ϕ respectively with a fixed directions then:

$$\frac{\sin\theta}{\sin\phi} \cdot \frac{d\,\theta}{d\,\phi} = \pm\frac{k}{\tau}$$

4) Prove that generators of a devlopable surface are tangents to curve.

5) On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the section by the planes z = constant.

6) Find the value of :

(i) First curvature

(ii) Gaussion curvature

at any point of right helicoids $x = u\cos\theta, y = \sin\theta, z = c\theta$

7) Prove that a curve on sphere is a geodesies if and only if it is a great circle.

8) A covariant tensor of first order has components $xy, 2y - z^2, xz$ in rectangular coordinates. Determine its covariant components in spherical polar coordinate.

9) Calculate the Christoffel symbols corresponding to metric $ds^2 = (dx^1)^2 + G\,(x^1, x^2)\,(dx^2)^2$ where G is a function of x^1 and x^2.

Section - C 2 x 16 = 32

Note : Section 'C' contain 04 Long answer type questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) (i) Prove that the necessary and sufficient condition for a space V_N to be flat is that the Riemann-Christoffel tensor be Identically Zero i.e. $R^{\alpha}_{ijk} = 0$.

(ii) If surface of sphere is a two dimensional Riemannian space. Compute the christoffel symbols.

11) (i) Prove that an entity whose inner product with an arbitrary tension is a tensor is itself a tensor.

(ii) Prove that on the surface $z = f(x, y)$ (Monge's form) the equation of asymptotic lines are:

$rdx^2 + 2s\,dx\,dy + t\,dy^2 = 0$

12) (i) Show that the vector B^i of variable magnitude suffers a parallel displacement along a curve C if and only if:

$$(B^l B^i_{,j} - B^i B^l_{,j}) \frac{dx^i}{ds} = 0$$

(ii) From the Gauss characteristics equation deduce that, when the parametric curves are orthogonal:

$$k = \frac{1}{\sqrt{EG}} \left[\frac{\partial}{\partial u} \left(\frac{1}{E} \frac{\sqrt{G}}{\partial u} \right) + \frac{\partial}{\partial v} \left(\frac{1}{\sqrt{G}} \frac{\partial \sqrt{E}}{\partial v} \right) \right]$$

13) (i) State and prove uniquennces theorem for space curves.

(ii) Prove that the torsion of the two Bertrand curves have the same sign and their product is constant.
