MAMT-02/MSCMT-02/MAT-102

June - Examination 2026

M.A./M.Sc. (Previous) Examination

MATHEMATICS

Real Analysis and Topology

Paper : MAMT-02/MSCMT-02/MAT-102

[*Time: **3** Hours*] [*Maximum Marks: **80***]

Note :- The question paper is divided into three Sections A, B and C. Write answers as per the given instructions in each section. Use of non-programmable scientific calculator is allowed in this paper.

Section-A **8×2=16**

(Very Short Answer Type Questions)

Note :- Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries **2** marks.

1. (i) Define σ ring.

(ii) Define directed set.

(iii) State Weierstrass approximation theorem.

(iv) State Minkowski's inequality.

(v) Write the necessary and sufficient conditions for a bounded function *f* defined on the interval [a, b] to be L-integrable.

(vi) Define Hilbert space.

(vii) Define Nets.

(viii) Define compact topological spaces.

Section-B

4×8=32

(Short Answer Type Questions)

Note :- Answer **any four** questions. You have to delimit your each answer maximum up to **200** words. Each question carries **8** marks.

2. Let $\{E_n\}$ be a countable collection of sets of real numbers, then show that -

$$m^*\left(\bigcup_n E_n\right) \leq \sum_n m^*(E_n)$$

3. Prove that the sets ϕ and R are measurable sets.

4. If a function is summable on E, then show that it is finite almost everywhere on E.

5. Prove that every complete orthonormal system satisfies Parseval's identity.

6. State and prove Holder's inequality.

7. Prove that for a subset A of a topological space $(X, \tau), \overline{A} = A \cup A'$.

8. Show that regularity is a topological property.

9. Prove that every filter F on a non-empty set X is contained in an ultra-filter on X.

Section-C

2×16=32

(Long Answer Type Questions)

Note :- Answer **any two** questions. You have to delimit your each answer maximum up to **500** words. Each question carries **16** marks.

10. Let $< f_n >$ be a sequence of bounded measurable functions defined on a set E of finite measure. If there exists a positive number M such that $|f_n(x)| \leq M$ for all $n \in N$ and for all $x \in E$ and if $< f_n >$ converges in measure to a bounded measurable function f on E, then prove that –

$$\lim_{n\to\infty} \int_E f_n(x)dx = \int_E f(x)dx$$

11. Prove that every interval is measurable.

12. Prove that union of arbitrary family of connected subset of a topological space is connected if the family is with non-empty intersection.

13. State and prove Riesz-Fisher theorem.
