MAMT-09/MSCMT-09/MAT-103

June - Examination 2026

M.A./M.Sc. (Previous/Final) Examination

MATHEMATICS

Integral Transforms and Integral Equations

Paper : MAMT-09/MSCMT-09/MAT-103

[*Time: **3** Hours*] [*Maximum Marks: **80***]

Note :- The question paper is divided into three Sections A, B and C. Write answers as per the given instructions in each section. Use of non-programmable scientific calculator is allowed in this paper.

Section-A **8×2=16**

(Very Short Answer Type Questions)

Note :- Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries **2** marks.

1. (i) Find the Laplace transform of $e^{3t} + 4\cos 2t$.

   (ii) Find the Inverse Laplace transform of $\frac{3}{s-4} - \frac{s}{s^2+25}$.

   (iii) State Parseval's identity for Fourier transform.

   (iv) Write relationship between Fourier transform and Laplace transform.

   (v) State Faltung theorem for the Mellin transform.

   (vi) Define resolvent kernel.

   (vii) Define singular integral equation.

   (viii) Define Abel Integral equation.

Section-B **4×8=32**

(Short Answer Type Questions)

Note :- Answer **any four** questions. You have to delimit your each answer maximum up to **200** words. Each question carries **8** marks.

2. Find the Laplace transform of $\sin\sqrt{t}$.

3. Find the Inverse Laplace transform of $e^{-2s/3}\frac{s}{s^2+4}$.

4. Find the Fourier transform of the function -

$$f(x)=\begin{cases} -(1+x), & -1 \le x \le 0 \\ x-1, & 0 < x \le 1 \\ 0, & |x|>1 \end{cases}$$

5. State and prove convolution theorem for Mellin Transform.

6. Show that the characteristic numbers of a symmetric kernel are real.

7. Using Hilbert-Schmidt theorem, find the solution of the symmetric integral equation –

$$g(x)=1+x^2+\frac{3}{2}\int_{-1}^{1}(xt+x^2t^2)g(t)dt$$

8. Prove that the characteristic numbers of a symmetric kernel are real.

9. Find the resolvent kernel and solution of -

$$g(x)=f(x)+\lambda\int_0^1(x+y)\,g(t)dt$$

Section-C **2×16=32**

(Long Answer Type Questions)

Note :- Answer **any two** questions. You have to delimit your each answer maximum up to **500** words. Each question carries **16** marks.

10. Solve - $\frac{d^4y}{dx^4}-y=1$

Subject to the conditions $y(0) = y'(0) = y''(0) = y'''(0) = 0$.

11. Find the Fourier transform of $(x)=\begin{cases}1 & |x|<a\\ 0 & |x|>a\end{cases}$ and hence evaluate -

$$\int_{-\infty}^{\infty}\frac{\sin sa\cos sa}{s}ds$$

12. Find the Eigen values and Eigen functions of the homogeneous integral equation -

$$g(x)=\lambda\int_0^1 K(x,t)g(t)dt,$$

where :

$$K(x,t)=\begin{cases}x(t-1), 0\le x\le t\\ t(x-1), t\le x\le 1\end{cases}$$

13. Show that the integral equation -

$$g(x)=f(x)+\frac{1}{\pi}\int_0^{2\pi}\sin(x+t)\,g(t)dt$$

possesses no solution for $f(x) = x$, but that it possesses infinitely many solution when $f(x) = 1$.
