MAMT-01/MSCMT-01/MAT-101

June - Examination 2026

M.A./M.Sc. (Previous) Examination

MATHEMATICS

Advanced Algebra

Paper : MAMT-01/MSCMT-01/MAT-101

[*Time: **3** Hours*] [*Maximum Marks: **80***]

Note :- The question paper is divided into three Sections A, B and C. Write answers as per the given instructions in each section. Use of non-programmable Scientific Calculator is allowed in this paper.

Section-A **8×2=16**

(Very Short Answer Type Questions)

Note :- Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries **2** marks.

1. (i) Define Conjugate elements.
   (ii) Define Commutators.
   (iii) Define Composition Series.
   (iv) Define prime elements in Integral domain.
   (v) Define rank of a Linear transformation.
   (vi) Define Separable extension.
   (vii) State Pythagoras Theorem.
   (viii) Define Adjoint operator.

Section-B **4×8=32**

(Short Answer Type Questions)

Note :- Answer **any four** questions. You have to delimit your each answer maximum up to **200** words. Each question carries **8** marks.

2. If A is a Sub-group of a group G and $N_G(A)$ is a normalizer of A in G, then show that A is normal in $N_G(A)$ and $N_G(A)$ is the largest sub-group of G in which A is normal.

3. Show that a group G is abelian if and only if the derived sub-group $G^1=\{e\}$, e being the identity of G.

4. Show that a finite group G is solvable if and only if G has a composition series whose factors are cyclic of prime order.

5. Let M be an R-module and let N be a sub module of M. Then show that the additive abelian quotient group M/N can be made into an R-module by defining scalar multiplication as -

$$r\,(N + x) = N + r\,x \;\forall\; r \in R\,,\; N + x \in M/N$$

6. If K/F is a finite field extension then show that it is an algebraic extension.

7. Let $A = [a_{ij}]$ be any $n \times n$ matrix over a field F, then show that -

$$\det(A) = \sum_{\sigma \in S_n} \in (\sigma)\; a_{\sigma(1)_1}\, a_{\sigma(2)_2} \ldots\ldots\; a_{\sigma(n)_n}$$

8. Show that an orthogonal set $A = \{x_i\}_{i=1}^{n}$ of nonzero vectors in an inner product space V is linearly independent.

9. Let V be a finite dimensional inner product space and W be its subspace. Then show that V is the direct sum of W and $W^{\perp}$ i.e. $V = W + W^{\perp}$

Section-C **2×16=32**

(Long Answer Type Questions)

Note :- Answer **any two** questions. You have to delimit your each answer maximum up to **500** words. Each question carries **16** marks.

10. Let G_1 and G_2 be groups. If N_i is normal in G_i, i = 1, 2, then show that $N_1 \times N_2$ is normal in $G_1 \times G_2$ and $(G_1 \times G_2) / (N_1 \times N_2) \cong (G_1/N_1) \times (G_2/N_2)$.

11. State and prove Unique Factorization Theorem.

12. Let K be a finite algebraic extension of a field F. Then show that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

13. If $\{u_1, u_2, \ldots\ldots, u_n\}$ is any finite orthonormal set in an inner product space V and ν is any vector in V, then show that -

$\sum_{i=0}^{n} |< \nu, u_i >|^2 \leq \|\nu\|^2$.

Further equality holds if and only if $\{u_1, u_2 \ldots, u_n\}$ is a basis of V.
