MAT-401/MAMT-06/MSCMT-06

June - Examination 2026

M.A./M.Sc. (Final) Examination

MATHEMATICS

Analysis and Advanced Calculus

Paper : MAT-401/MAMT-06/MSCMT-06

[*Time: **3** Hours*] [*Maximum Marks: **80***]

Note :- The question paper is divided into three Sections A, B and C. Write answers as per the given instructions in each section. Use of non-programmable scientific calculator is allowed in this paper.

Section-A **8×2=16**

(Very Short Answer Type Questions)

Note :- Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries **2** marks.

1. (i) Explain Weak convergence.
   (ii) State Open Mapping Theorem.
   (iii) State Hahn-Banach Theorem.
   (iv) If X be a complex inner product space and x, y, z ∈ X, then prove that $(x,\ \beta y + \gamma z) = \bar{\beta}(x,\ y) + \bar{\gamma}(x,\ z)$ where $\beta,\ \gamma \in C$.
   (v) Define Ortho-normal set.
   (vi) Define Self-adjoint operator.
   (vii) Define Perpendicular projection.
   (viii) Define Regulated function.

Section-B **4×8=32**

(Short Answer Type Questions)

Note :- Answer **any four** questions. You have to delimit your each answer maximum up to **200** words. Each question carries **8** marks.

2. Prove that every compact subset of a normed linear space is complete and bounded.
3. Prove that closed convex subset K of a Hilbert Space H contains a unique vectors of smallest norm.
4. If P is a projection on a Hilbert space H with range M and null space N, then prove that $M \perp N$ if and only if P is self adjoint, and in this case $N = M^{\perp}$.
5. If M be a proper closed linear subspace of a Hilbert space H, then prove that there exists a nonzero vector z_0 in H such that $z_0 \perp M$.
6. If T_1 and T_2 are normal operators on H with the property that either commutes with adjoint of the other, then prove that $T_1 + T_2$ and $T_1.T_2$ are normal.

7. If T is normal operator on a Hilbert space H, then prove that eigenspaces of T are pairwise orthogonal.

8. If X and Y be any two Banach spaces over the same field K of scalars and V be an open subset of X. If $f : V \to Y$ be continuous function and u, v be any two distinct points of V such that $[u, v] \subset V$ and f is differentiable in [u, v], then prove that $\|f(v) - f(u)\| \leq \|v - u\| \sup\{\|Df(x)\| : x \in [u, v]\}$.

9. If I be an interval in R, W a closed set of a Banach space X and $g : I \times W \to X$ be a continuous function c-Lipsehitz in $x \in X$. Let $(s, x_0) \in I \times W$ for given $r > 0$ and $f : I \to X$ be an r-approximate solution of differential equation $\frac{dx}{dt} = g(x, t)$ such that $f(s) = x_0$, then prove that there exists in I an exact solution $\phi : I \to X$ of the differential equation such that $\phi(s) = x_0$.

Section-C

2×16=32

(Long Answer Type Questions)

Note :- Answer **any two** questions. You have to delimit your each answer maximum up to **500** words. Each question carries **16** marks.

10. State and prove Risez Representation Theorem.

11. (a) Show that the linear spaces R (real) and C (complex) are normed linear spaces under the norm $\|x\| = |x|$, $x \in C$. Also show that these spaces are Banach spaces.

    (b) If M be a closed linear subspace of a normed linear space N and x_0 be a vector in N, but not in M with the property that the distance from x_0 to M i.e. $d(x_0, M) = d > 0$, then prove that there exists a bounded linear functional $F \in N$ such that $\|F\| = 1$, $F(x_0) = d$, $F(x) = 0 \ \forall \ x \in M$ i.e. $F(M) = \{0\}$.

12. If X be a Banach space over the field K of scalars and if f:[a, b] → X and g:[a, b] → R be continuous and differentiable functions such that $\|Df(t)\| \leq Dg(t)$ at each point $t \in (a, b)$, then prove that $\|f(b) - f(a)\| \leq g(b) - g(a)$.

13. If f be a functional defined on a linear subspace M of a normed linear space N, $x_0 \notin M$ and $M_0 = [M \cup \{x_0\}] = \{x + ax_0 : x \in M, \text{‘a’ is real}\}$ is the linear subspace spanned by M and x_0, then prove that f can be extended to a functional f_0 defined on M_0 that $\|f_0\| = \|f\|$.
