MAT-302/MAMT-04/MSCMT-04

June - Examination 2026

M.A./M.Sc. (Previous/Final) Examination

MATHEMATICS

Differential Geometry and Tensor

Paper : MAT-302/MAMT-04/MSCMT-04

[*Time: **3** Hours*] [*Maximum Marks: **80***]

Note :- The question paper is divided into three Sections A, B and C. Write answers as per the given instructions in each section.

Section-A **8×2=16**

(Very Short Answer Type Questions)

Note :- Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries **2** marks.

1. (i) Write the equation of osculating plane in Cartesian coordinates.

   (ii) Define Bertrand curves.

   (iii) Define ruled surface.

   (iv) Write the Weingarten equations.

   (v) Write the statement of Beltrami-Enneper theorem.

   (vi) Define Kronecker delta.

   (vii) What is Euler's condition?

   (viii) Define Einstein tensor.

Section-B **4×8=32**

(Short Answer Type Questions)

Note :- Answer **any four** questions. You have to delimit your each answer maximum up to **200** words. Each question carries **8** marks.

2. Find the inflexional tangents at (x, y, z) on the surface $y^2z = 4ax$.

3. Find the radii of curvature and torsion of the helix -

   $x = a\cos\theta$, $y = a\sin\theta$, $z = a\,\theta\tan\alpha$

4. Find the developable surface which passes through the curves $y^2 = 4ax$, $z = 0$ and

   $y^2 = 4bz$, $x = 0$.

5. Derive the relation between three fundamental forms.

6. Prove that a curve on sphere is a geodesic if and only if it is a great circle.

7. Calculate the Christoffel symbols corresponding to the metric $ds^2 = (dx^1)^2 + G(x^1, x^2)(dx^2)^2$ where G is a function of x^1 and x^2.

8. Prove that, if A^i is a contravariant vector, then div $A^i = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i} \left(A^i \sqrt{g}\right)$.

9. Prove that the geodesic is an auto parallel curve.

Section-C **2×16=32**

(Long Answer Type Questions)

Note :- Answer **any two** questions. You have to delimit your each answer maximum up to **500** words. Each question carries **16** marks.

10. Derive Serret-Frenet formulae.

11. Prove that the metric of a surface is invariant under parametric transformation.

12. Find the principal radii at the origin of the surface $2z = 5x^2 + 4xy + 2y^2$. Find also the radius of curvature of the section $x = y$.

13. Show that the metric of a Euclidean space, referred to spherical coordinates is given by -

$$ds^2 = (dr)^2 + (rd\theta)^2 + (r \sin\theta \, d\phi)^2$$

Determine its metric tensor and conjugate metric tensor.
