MAT-201/MAMT-03/MSCMT-03

June - Examination 2026

M.A./M.Sc. (Previous) Examination

MATHEMATICS

Differential Equations, Calculus of Variations and Special Functions

Paper : MAT-201/MAMT-03/MSCMT-03

[*Time:* ***3*** *Hours*] [*Maximum Marks:* ***80***]

Note :- The question paper is divided into three Sections A, B and C. Write answers as per the given instructions in each section.

Section-A **8×2=16**

(Very Short Answer Type Questions)

Note :- Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries **2** marks.

1. (i) Define Riccati's equation.

   (ii) Write the necessary and sufficient condition for the Total Differential Equation Pdx + Qdy + Rdz = 0 to be integrable.

   (iii) Write Monge's subsidiary equation for Rr + Ss + Tt = V.

   (iv) Define Functionals.

   (v) Define Eigen values and Eigen functions.

   (vi) Write Legendre's Equation.

   (vii) Write the generating function for Bessel function Jn(x).

   (viii) Classify the following differential equation -

$$4\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial^2 u}{\partial x\,\partial y} + 9\frac{\partial^2 u}{\partial y^2} = 0$$

Section-B **4×8=32**

(Short Answer Type Questions)

Note :- Answer **any four** questions. You have to delimit your each answer maximum up to **200** words. Each question carries **8** marks.

2. Find the general solution of the Riccati's Equation $dy/dx = 2 - 2y + y^2$ whose one particular solution is (1 + tanx).

3. Using general method, solve total differential equation -
   $3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})dz = 0$

4. Solve the heat conduction equation -

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

   given that u = 0, when $t \to \infty$, x = 0 , x = l

5. Find the extremals of the functional -

$$I[y(x), z(x)] = \int_0^{\pi/2} (y'^2 + z'^2 + 2yz)dx$$

with the boundary conditions

$y_{(0)} = 0$, $y_{(\pi/2)} = -1$; $z_{(0)} = 0$, $z_{(\pi/2)} = 1$

6. Find the extremals of the functional -

$$I = \int_0^1 (1 + y''^2)dx$$

subject to the conditions

$y_{(0)} = 0$, $y'_{(0)} = 1$, $y_{(1)} = 1$, $y'_{(1)} = 1$

7. If $|z| < 1$ and $\left|\frac{z}{1-z}\right| < 1$, then show that

$${}_2F_1(\alpha, \beta : \gamma : z) = (1 - z)^{-\beta}\, {}_2F_1(\gamma - \alpha, \beta;\ \gamma;\ \tfrac{z}{z-1})$$

8. Prove that -

$$L_3(x) = \frac{1}{3!}[6 - 18x + 9x^2 - x^3]$$

9. Solve : $2s + (rt - s^2) = 1$

Section-C 2×16=32

(Long Answer Type Questions)

Note :- Answer **any two** questions. You have to delimit your each answer maximum up to **500** words. Each question carries **16** marks.

10. Reduce the equation -

$$\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x\, \partial y} + 3\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 2$$

to canonical form and solve it.

11. Solve -

$$2x(1 - x)\frac{d^2y}{dx^2} + (1 - x)\frac{dy}{dx} + 3y = 0$$

in power series.

12. Find the Eigen values and Eigen functions of the Sturm-Liouville problem

$$\frac{d}{dx}\left[x\frac{dy}{dx}\right] + \frac{\lambda}{x}\, y = 0$$

with $y_{(1)} = 0$, $y_{(e^{\pi})} = 0$

13. Prove that -

$$\int_{-\infty}^{\infty} e^{-x^2} H_m(x) H_n(x)dx = \begin{cases} 0, & m \neq n \\ 2^n n!\sqrt{\pi}, & m = n \end{cases}$$
