# वर्धमान महावीर खुला विश्वविद्यालय, कोटा <br> रावतभाटा रोड , कोटा 324021 (राजस्थान) <br> फोन: - 0744-2470615, कैक्स: - 0744-2472525 <br> Visit us at: www.vmou.ac.in 

## INTERNAL ASSIGNMENT



## M.A. /M.Sc. Previous (Mathematics)

प्रिय छात्र,
आपको M.A. /M.Sc. Previous (Mathematics) के पाठ्यक्रम के विभिन्न प्रश्न पत्रों के सत्रीय कार्य दिए जा रहे है। आपको प्रत्येक प्रश्न पत्र के दिए गए सत्रीय कार्य करने हैं। इन्हें पूरा करके आप निर्धारित अंतिम तिथि से पूर्व अपने क्षेत्रीय केंद्र/अध्ययन केंद्र (जहाँ पर आपने प्रवेश लिया है) पर स्वयं अथवा पंजीकृत डाक से आवश्यक रूप से भिजवा दें। प्रत्येक सत्रीय कार्य 20 अंकों का हैं। इन प्राप्तांको को आपकी सत्रांत परीक्षा के अंकों में जोड़ा जायेगा। सत्रीय कार्य स्वयं की हस्तलिपि में करें। सत्रीय कार्यों का पुनर्मूल्यांकन नहीं होता है और न ही इन्हें सुधारने हेतु दु बारा स्वीकार किया जाता हैं। अतः आप एक बार में ही सही उत्तर लिखें। आप संलग्न निर्धारित प्रपत्र पर वांछित सूचना भरकर सत्रीय कार्य के साथ संलग्न करें।

## M.A. /M.Sc. Previous (Mathematics) 2014-15

## Internal Assignment

1. पाठ्यक्रम कोड (Course Code).

2. पाठ्यक्रम का नाम
3. स्कॉलर संख्या (Scholar No.)

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4. छात्र का नाम

Name of Student (in capital letters)
5. पिता का नाम

Name of Father (in capital letters)

6. पत्र व्यवहार का पता $\qquad$
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Address for Corresponding

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7. अध्ययन केंद्र का नाम

Name of Study Centre

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8. क्षेत्रीय केंद्र (Regional Centre).

| Ajmer | Bikaner | Jaipur | Jodhpur | Kota | Udaipur |
| :--- | :--- | :--- | :--- | :--- | :--- |

जमा करवाने का दिनांक (Date of Submission) $\qquad$

Internal Assignment-2014<br>Program Name M.Sc. / M.A. (Mathematics)<br>Paper Code - MT- 01(Advanced Algebra)<br>M.Sc. / M.A. (Previous)<br>Max. Marks 20

Note:- The Internal Assignment has been divided into three sections A, B and C. Write Answer as per the given instructions.

## Section-A <br> (Very Short Answer Type Questions)

Note :- Answer all questions. As per the nature of the question you delimit your answer in one word, one sentence or maximum up to 30 words. Each question carries 1 (one) mark.

$$
4 \times 1=4
$$

1. If H and K are two sub groups of G such that G is an internal direct product of H and K. Then find $H \cap K$.
2. Write class equation for the finite group.
3. Define normal extension of a field.
4. Write Schwartz inequality.

## Section - B

(Short Answer Questions)
Note :- Answer any two questions . Each answer should be given in 200 words. Each question carries 4 marks .

$$
2 \times 4=8
$$

5. Let V and $V^{\prime}$ be inner product space and $t: V \rightarrow V^{\prime}$ be an orthogonal linear transformation. Then prove the following :
(a) $t$ is monomorphism.
(b) If $\left\{u_{1}, u_{2}, \ldots \ldots u_{n}\right\}$ is orthonormal then $\left\{t\left(u_{1}\right), t\left(u_{2}\right), \ldots \ldots t\left(u_{n}\right)\right\}$ is orthonormal in $V^{\prime}$.
6. If $B=\left\{u_{1}, u_{2}, \ldots \ldots . u_{n}\right\}$ an orthonormal basis of an inner product space V and $v \in V$ be any arbitrary vector. Then prove that the coordinates of $v$ relative to the basis B of v are $<v, u_{i}>, i=1,2, \ldots . . n$ and

$$
\|v\|^{2}=\sum_{i=1}^{n}\left|v, v_{i}\right|^{2}
$$

7. Let V be a finite dimensional vector space over a field F and $t: V \rightarrow V$ be a linear transformation. Suppose that $V_{1}, V_{2}, \ldots . V n$ are distinct eigen vectors of $t$ corresponding to distinct eigenvalues $\lambda_{1}, \lambda_{1}, \ldots \ldots \ldots . \lambda_{n}$. Then prove that $\left\{V_{1}, V_{2}, \ldots . V n\right\}$ is a linearly independent set.
8. Let $F$ be a field and let $f(x)$ be an irreducible polynomial in $f(x)$. The prove that $f(x)$ has a multiple root in some field extension if and only if $f^{\prime}(x)=0$

## Section - C <br> (Long Answer Questions)

Note :- Answer any one question. Each answer should be given in 800 words. Each question carries 08 marks.
$1 \times 8=8$
9. Let $M_{1}$ and $M_{2}$ are submodule of an R-module M. Then prove that:

$$
\frac{M_{1}+M_{2}}{M_{2}} \cong \frac{M_{1}}{M_{1} \cap M_{2}}
$$

10. Let V and $V^{\prime}$ be any two finite dimensional vector space over the same field F . Then prove that the vector space $\operatorname{Hom}\left(\mathrm{V}, V^{\prime}\right)$ of all linear transformation of V to $V^{\prime}$ is also finite dimensional and dim $\operatorname{Hom}\left(\mathrm{V}, V^{\prime}\right)=\operatorname{dim} \mathrm{V} \times \operatorname{dim} V^{\prime}$.

## Internal Assignment-2014

Program Name M.Sc. / M.A. (Mathematics) Paper Code - MT- 02(Real Analysis and Topology)<br>M.Sc. / M.A. (Previous)<br>Max. Marks 20

Note:- The Internal Assignment has been divided into three sections A, B and C. Write Answer as per the given instructions .

## Section-A

## (Very Short Answer Type Questions)

Note :- Answer all questions. As per the nature of the question you delimit your answer in one word, one sentence or maximum up to 30 words. Each question carries 1 (one ) mark.

$$
4 \times 1=4
$$

1. If $\left\langle f_{n}\right\rangle$ is a convergent sequence of measurable functions defined on a measurable set E , then the limit function of $<f_{n}>$ is:
2. Define a $\sigma$-ring.
3. The one point compactification of the set complex numbers is called:
4. What is Bolazano -weirstrass property?

## Section - B <br> (Short Answer Questions)

Note :- Answer any two questions . Each answer should be given in 200 words. Each question carries 4 marks . $\mathbf{2 \times 4}=\mathbf{8}$
5. Show that every bounded measurable functions $f$ defined on a measurable set $E$ is Lintegrable on E .
6. Let $<f_{n}>$ be a sequence of non-negative measurable functions. If $\lim _{n \rightarrow \infty} f_{n}\left(x_{0}\right)=$ $f\left(x_{0}\right)$ at a point $x_{0}$ then Prove that for each $m \in N$

$$
\lim _{n \rightarrow \infty}\left[f_{n}\left(x_{0}\right)\right]_{m}=\left[f\left(x_{0}\right)\right]_{m}
$$

7. Show that a sequence of functions in $L^{p}$-space has a unique limit.
8. Show that every $T_{3}$-space is a $T_{2}$-space.
9. 

## Section - C <br> (Long Answer Questions)

Note :- Answer any one question. Each answer should be given in 800 words. Each question carries 08 marks.

$$
1 \times 8=8
$$

10. State and prove Parseval's identity in $L_{2}$.
11. Show that one point compactification of set of rational numbers $Q$ is not Hausdorff.

## Internal Assignment-2014

Program Name M.Sc. / M.A. (Mathematics)<br>Paper Code - MT- 03<br>(Differential Equations, Calculus of Variations<br>and Special Functions)<br>M.Sc. / M.A. (Previous)<br>Max. Marks 20

Note:- The Internal Assignment has been divided into three sections A, B and C. Write Answer as per the given instructions.

## Section -A

## (Very Short Answer Type Questions)

Note :- Answer all questions. As per the nature of the question you delimit your answer in one word, one sentence or maximum up to 30 words. Each question carries 1 (one ) mark.

$$
4 \times 1=4
$$

1. Write down the Riccati's equation.
2. Write down Euler-Largange equation.
3. If $|z|<1$ and $\left|\frac{z}{1-z}\right|<1$ then write the value of $2 F_{1}\left(a, b ; c ; \frac{1}{2}\right)$.
4. Write down the Legendre equation.

## Section - B

(Short Answer Questions)
Note :- Answer any two questions . Each answer should be given in 200 words. Each question carries 4 marks.

$$
2 \times 4=8
$$

5. Prove that :

$$
\int_{x}^{\infty} e^{-t} L_{m}(x) L_{n}(x) d x= \begin{cases}0 & \text { if } m \neq n \\ 1 & \text { if } m=n\end{cases}
$$

6. Prove that:
$(2 n+1)\left(x^{2}-1\right) P^{\prime}{ }_{n}=n(n+1)\left(P_{n+1}-P_{n-1}\right)$
and hence deduce that
$\int_{-1}^{1}\left(x^{2}-1\right) P_{n+1}(x) P_{n}^{\prime}(x) d x=\frac{2 n(n+1)}{(2 n+1)(2 n+3)}$
7. If m is a positive integer and $|x|>1$ show that:

$$
2 f_{1}\left(m+\frac{1}{2}, m+\frac{2}{2} ; 1 ; \frac{1}{x^{2}}\right)=\frac{(-1)^{m} x^{m+1}}{\llcorner m} \frac{d^{m}}{d x^{m}}\left\{\frac{1}{\sqrt{x^{2}+1}}\right\}
$$

8. Prove that (Kummer's Theorem)

$$
2 f_{1}(a, b ; 1-a+b ;-1)=\frac{\digamma(1-a+b) \Gamma\left(1+\frac{b}{2}\right)}{\Gamma(1+b) \digamma\left(1+\frac{b}{2}-a\right)}
$$

## Section - C

## (Long Answer Questions)

Note :- Answer any one question. Each answer should be given in 800 words. Each question carries 08 marks.
$1 \times 8=8$
9. (a) Solve the following strem Liouville problem.
$y^{\prime \prime}+\lambda y=0, \quad y^{\prime}(-\pi)=0, \quad y^{\prime}(\pi)=0$
(b) Find the externals of the functional:
$I(y, z)=\int_{0}^{\pi / 2}\left[y^{2}+z^{2}+2 y z\right] d t$
With the boundary condition $y(0)=0, y\left(-\frac{\pi}{2}\right)=-1, z(0)=0, x\left(\frac{\pi}{2}\right)=1$
10. (a) A tightly stretched string which has fixed end points $x=0$ and $x=l$ is initially in a position given by $y=k \sin ^{3}\left(\frac{\pi x}{e}\right)$. It is released from rest from this position. Find the displacement $y(x, t)$.
(b) Reduce $\frac{\partial^{2} z}{\partial x^{2}}=x^{2} \frac{\partial^{2} z}{\partial y^{2}}$ to canonical form.

## Internal Assignment-2014

## Program Name M.Sc. / M.A. (Mathematics)

 Paper Code - MT- 04(Differential Geometry \& Tensors)M.Sc. / M.A. (Previous)

## Max. Marks 20

Note:- The Internal Assignment has been divided into three sections A, B and C. Write Answer as per the given instructions.

## Section-A

## (Very Short Answer Type Questions)

Note :- Answer all questions. As per the nature of the question you delimit your answer in one word, one sentence or maximum up to 30 words. Each question carries 1 (one ) mark. $4 \times 1=4$

1. Write down the equation of tangent line to a curve at a given point.
2. Define the oscillating circle.
3. Write down the formula to the curvature of evolute.
4. State Mennier's theorem.

## Section - B <br> (Short Answer Questions)

Note :- Answer any two questions . Each answer should be given in 200 words. Each question carries 4 marks . $\mathbf{2 x 4}=\mathbf{8}$
5. Determine the function $\mathrm{f}(0)$ so that $x=\cos \theta, y=a \sin v, z=f(v)$ shall be a plane curve.
6. State and prove schur's theorem.
7. If the intrinsic Derivative of a vector $A^{i}$ along a curve C vanish at every point of the curve, then show that the magnitude of the vector $A^{i}$ is constant along the curve.
8. A Wuartant tensor of first order has components $x y, 2 y-z^{2}, x z$ in rectangular coordinates. Determine its covariant components in spherical coordinate.

## Section-C <br> (Long Answer Questions)

Note :- Answer any one question. Each answer should be given in 800 words. Each question carries 08 marks.
$1 \times 8=8$
9. (a) Suppose that a tangent plane to the ellipsoid $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}+\frac{z^{2}}{c^{2}}=1$ meetsthe coordinate axes in points $\mathrm{P}, \mathrm{Q}, \mathrm{R}$. Prove that the envelope of the sphere OPQR is $(a x)^{2 / 3}+$ $(\text { by })^{2 / 3}+(C Z)^{2 / 3}=\left(x^{2}+y^{2}+z^{2}\right)^{2 / 3}$ Where 0 is the origin.
(b) Find the angle between two tangential direction on the surface in the terms of direction ratio.
10. (a) Prove that the metric of a surface is invariant under parametric transformation.
(b) State and prove second fundamental theorem.

Internal Assignment-2014<br>Program Name M.Sc. / M.A. (Mathematics)<br>Paper Code - MT- 05(Mechanics)<br>M.Sc. / M.A. (Previous)<br>Max. Marks 20

Note:- The Internal Assignment has been divided into three sections A, B and C. Write Answer as per the given instructions .

## Section -A

## (Very Short Answer Type Questions)

Note :- Answer all questions. As per the nature of the question you delimit your answer in one word, one sentence or maximum up to 30 words. Each question carries 1 (one ) mark.

$$
4 \times 1=4
$$

1. Write the Newton's second Law of motion.
2. Write the complex potential when a source of strength $m$ and a sink of strength (-m) at a point $(a, 0)$ and $(0, a)$ respectively.
3. Define a doublet.
4. Write the bernoulli's equation for the unsteady-irrotational motion of an incompressible fluid.

## Section - B

(Short Answer Questions)
Note :- Answer any two questions . Each answer should be given in 200 words. Each question carries 4 marks .

$$
2 \times 4=8
$$

5. State and prove $D^{\prime}$ Alembert's principle.
6. A uniform rod of mass $m$ is placed at right angles to a smooth plane of inclination $\alpha$ with one end in contact with it. The rod is then released. Show that when the inclination to the plane is $\phi$, the reaction of the plane will be $\left\{\frac{3(1-\sin \phi)^{2}+1}{\left(1+3 \cos ^{2} \phi\right)^{2}}\right\} m g \cos \alpha$.
7. A dice in the form of a portion of parabola bounded by its latus rectum and its axis has its vertex A fixed and is stuck by a blow through the end of its latus rectum perpendicular to its plane. Show that the dice starts revolving about a line through A inclined at an angle $\tan ^{-1}\left(\frac{14}{25}\right)$ to the axis.
8. Use Lagrange's equations to find the equation of motion of a simple pendulum.

## Section - C <br> (Long Answer Questions)

Note :- Answer any one question. Each answer should be given in 800 words. Each question carries 08 marks.
$1 \times 8=8$
9. (a) Three equal uniform rods $\mathrm{AB}, \mathrm{BC}, \mathrm{DC}$ are smoothly jointed at B and C and the ends A and D are fastened to smooth fixed points whose distance a part is equal to the length of either rod. The frame being at rest in the from of a square. A blow I is given perpendicular to AB at its middle point and in the plane of the square. Show that the energy set up is $\frac{3 I^{2}}{40 \mathrm{~m}}$, where m is the mass of each rod. Find also the blows at the joints A and C.
(b) Deduce the Lagrange's equations from Hamilton's Principle.
10. (a) A mass of fluid is in motion such that the lines of motion lie on the surface of coaxial cylinders, show that the equation of continuity is $\frac{\partial P}{\partial t}+\frac{1}{r} \frac{\partial(P u)}{\partial \theta}+\frac{\partial(P v)}{\partial z}=0$. where $\mathrm{u}, \mathrm{v}$ are the velocity perpendicular and parallel to z .
(b) Find the Cauchy-Riemann equations in polar coordinates.

