

# वर्धमान महावीर खुला विश्वविद्यालय, कोटा

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## INTERNAL ASSIGNMENT



## M.A. /M.Sc. Final (Mathematics)

प्रिय छात्र,

आपको M.A. /M.Sc. Final (Mathematics) के पाठ्यक्रम के विभिन्न प्रश्न पत्रों के सत्रीय कार्य दिए जा रहे हैं। आपको प्रत्येक प्रश्न पत्र के दिए गए सत्रीय कार्य करने हैं। इन्हें पूरा करके आप निर्धारित अंतिम तिथि से पूर्व अपने क्षेत्रीय केंद्र/अध्ययन केंद्र (जहाँ पर आपने प्रवेश लिया है) पर स्वयं अथवा पंजीकृत डाक से आवश्यक रूप से भिजवा दें। प्रत्येक सत्रीय कार्य 20 अंकों का है। इन प्राप्तांको को आपकी सत्रांत परीक्षा के अंकों में जोड़ा जायेगा। सत्रीय कार्य स्वयं की हस्तलिपि में करें। सत्रीय कार्यों का पुनर्मूल्यांकन नहीं होता है और न ही इन्हें सुधारने हेतु दुबारा स्वीकार किया जाता है। अतः आप एक बार में ही सही उत्तर लिखें। आप संलग्न निर्धारित प्रपत्र पर वांछित सूचना भरकर सत्रीय कार्य के साथ संलग्न करें।

M.A. /M.Sc. Final (Mathematics) 2014-15

Internal Assignment

1. पाठ्यक्रम कोड (Course Code).....

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2. पाठ्यक्रम का नाम .....

3. स्कॉलर संख्या (Scholar No.).....

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4. छात्र का नाम .....

Name of Student (in capital letters)

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5. पिता का नाम .....

Name of Father (in capital letters)

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6. पत्र व्यवहार का पता .....

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Address for Corresponding


7. अध्ययन केंद्र का नाम .....

Name of Study Centre

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8. क्षेत्रीय केंद्र (Regional Centre).....

Ajmer	Bikaner	Jaipur	Jodhpur	Kota	Udaipur
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जमा करवाने का दिनांक (Date of Submission) .....

**Internal Assignment-2014**  
**Program Name M.Sc. / M.A. (Mathematics)**  
**Paper Code – MT- 06(Analysis and Advanced Calculus)**  
**M.Sc. / M.A. (Final)**  
**Max. Marks 20**

**Note:- The Internal Assignment has been divided into three sections A, B and C. Write Answer as per the given instructions .**

**Section –A**  
**(Very Short Answer Type Questions)**

Note :- Answer all questions . As per the nature of the question you delimit your answer in one word , one sentence or maximum up to 30 words. Each question carries 1 (one ) mark.

**4 x1 = 4**

1. Write integral solution of the differential equation  $\frac{dx}{dt} = g(t_1x)$
2. Define Directional derivative.
3. Write the condition on which an operator T on H becomes Isometric.
4. Write the Parseval's identity.

**Section – B**  
**(Short Answer Questions)**

**Note :-** Answer any two questions . Each answer should be given in 200 words. Each question carries 4 marks .

**2 x4 = 8**

5. If T be a linear transformation from a normed linear space into normed space  $N^1$ , then show that T is continuous either at every point or at no point of N.
6. Let B and  $B^1$  be Banach spaces. If T is a continuous linear transformation of B and  $B^1$ , then prove that T is an open mapping.
7. If x and y are any two vectors in a Hilbert space H, then prove  
 $\|(x + y)\|^2 + \|(x - y)\|^2 = 2(\|x\|^2 + \|y\|^2)$
8. If x is an eigenvector of T, then show that x cannot corresponding more than one eigenvalue of T.

**Section – C**  
**(Long Answer Questions)**

Note :- Answer any one question. Each answer should be given in 800 words. Each question carries 08 marks.

**1 x 8 = 8**

9. (a) State and prove Holder's inequality.  
(b) Let  $N$  and  $N^1$  be normed linear spaces over the same scalar field and let T be a linear transformation of  $N$  into  $N^1$ , then prove that T is bounded if it is continuous.
10. (a) State and prove the uniform boundedness theorem.  
(b) State and prove the Riesz Representation theorem.

# M.A./M.Sc. Maths Assignment 2014-15

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**Internal Assignment-2014**  
**Program Name M.Sc. / M.A. (Mathematics)**  
**Paper Code – MT- 07(Viscous Fluid Dynamics)**  
**M.Sc. / M.A. (Final)**  
**Max. Marks 20**

**Note:- The Internal Assignment has been divided into three sections A, B and C. Write Answer as per the given instructions .**

**Section –A**  
**(Very Short Answer Type Questions)**

Note :- Answer all questions . As per the nature of the question you delimit your answer in one word , one sentence or maximum up to 30 words. Each question carries 1 (one ) mark.

**4 x1 = 4**

1. Write down the energy equation in plane-couette flow with transpiration cooling.
2. What do you mean by porous boundaries?
3. What is stagnation point?
4. What is the coefficient of skin friction of plane Couette flow.

**Section – B**  
**(Short Answer Questions)**

**Note :-** Answer any two questions . Each answer should be given in 200 words. Each question carries 4 marks .

**2 x4 = 8**

5. The stress tensor at a point P is:

$$\sigma_{ij} = \begin{vmatrix} 7 & 0 & -2 \\ 0 & 5 & 0 \\ 2 & 0 & 4 \end{vmatrix}$$

Determine the stress vector on the plane at P whose unit normal is

$$\hat{n} = \frac{2\hat{i}}{3} - \frac{2}{3}\hat{j} + \frac{1\hat{k}}{3}$$

6. State law of conservation, define equation of continuity and write the eq<sup>n</sup> of continuity in Cartesian tensor notation.
7. What is the physical importance of non-dimension parameters. Discuss the Froude number.
8. Describe stream function for Oseen's flow.

**Section – C**  
**(Long Answer Questions)**

Note :- Answer any one question. Each answer should be given in 800 words. Each question carries 08 marks.

**1 x 8 = 8**

9. (a) Velocity field at the point is given by  $1 + 2y - 3z, 4 - 2x + 6z, 6 + 3x - 5y$   
Show that it represents a rigid body motion.  
(b) Define circulation show that the time rate of change of circulation in a closed circuit. Drawn in a viscous incompressible fluid under the action of conservative forces. Moving with fluid depends only on the kinematic viscosity and on the space rate of change of the vorticity components at the contour.
10. (a) Deduce fundamental equations of a viscous incompressible fluid with constant fluid properties in cylindrical polar coordinates.  
(b) State and prove Buckingham  $\pi$ -theorem.

# M.A./M.Sc. Maths Assignment 2014-15

**Internal Assignment-2014**  
**Program Name M.Sc. / M.A. (Mathematics)**  
**Paper Code – MT- 08(Numerical Analysis)**  
**M.Sc. / M.A. (Final)**  
**Max. Marks 20**

**Note:- The Internal Assignment has been divided into three sections A, B and C. Write Answer as per the given instructions .**

**Section –A**  
**(Very Short Answer Type Questions)**

**Note :- Answer all questions . As per the nature of the question you delimit your answer in one word , one sentence or maximum up to 30 words. Each question carries 1 (one ) mark.**

**4 x1 = 4**

1. If  $\lambda_1, \lambda_2, \lambda_3 \dots$  are the eigen values of square matrix A then what are the eigenvalues of  $A^{-1}$ .
2. Write down the remainder term of Taylor series?
3. Write down the error equation of Newton-Raphson extended formula?
4. What is the method of absolutely stable?

**Section – B**  
**(Short Answer Questions)**

**Note :- Answer any two questions . Each answer should be given in 200 words. Each question carries 4 marks .**

**2 x4 = 8**

5. Find a real root of the equation  $x^4 + 7x^3 + 24x^2 - 15 = 0$ , using Birge-Vieta method, perform two iterations.
6. Using the method of least-squares find a straight line that fits the following data:

x	71	68	73	69	67	65	66	67
y	69	72	70	70	68	67	68	64

Also find the value of y at  $x = 68.5$ .

7. Obtain a second degree polynomial approximation to the function:

$$f(x) = \frac{1}{1+x^2}, \quad x \in [1, 12]$$

Using Taylor series expansion about  $x = 1$ . Find a bound on the truncation error.

8. Evaluate  $y(1.5)$  by Adams-Bashfourth method of order four given that

$$\frac{dy}{dt} = t^2(1+y)$$

$$y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979, y(1.4) = 2.575$$

**Section – C**  
**(Long Answer Questions)**

**Note :- Answer any one question. Each answer should be given in 800 words. Each question carries 08 marks.**

**1 x 8 = 8**

9. Explain Rutishauser method & Using the Rutishauser method, find all the eigenvalues of the matrix.

$$A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$$

10. (a) Explain Aitken's  $\Delta^2$ -method to accelerate the convergence.  
(b) Solve the BVP by Numerical method.

$$\frac{d^2y}{dx^2} = x + y, \quad y(0) = 0, \quad y(1) = 0 \text{ with step size } h = \frac{1}{4}$$

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**Internal Assignment-2014**

**Program Name M.Sc. / M.A. (Mathematics)**

**Paper Code – MT- 09(Integral Transforms and Integral Equations)**

**M.Sc. / M.A. (Final)**

**Max. Marks 20**

**Note:- The Internal Assignment has been divided into three sections A, B and C. Write Answer as per the given instructions .**

**Section –A**

**(Very Short Answer Type Questions)**

Note :- Answer all questions . As per the nature of the question you delimit your answer in one word , one sentence or maximum up to 30 words. Each question carries 1 (one ) mark.

**4 x1 = 4**

1. Evaluate :  $L^{-1} \left\{ \frac{1}{(p-4)^5} + \frac{5}{(p-2)^2+5^2} + \frac{p+3}{(p+3)^2+6^2} \right\}$
2. State the mellin inversion theorem.
3. State Hilbert-Schmidt Theorem
4. State Fredholm's first fundamental theorem.

**Section – B**

**(Short Answer Questions)**

Note :- Answer any two questions . Each answer should be given in 200 words. Each question carries 4 marks .

**2 x4 = 8**

5. Solve the integral equation-

$$g(x) = f(x) + \lambda \int_{-1}^1 (xt + x^2t^2) g(t) dt$$

Also find its resolvent kernel.

6. Prove that the eigen values of a symmetric kernel are real.  
7. Prove that if n is a positive integer :

$$M \left\{ \left( x \frac{d}{dx} \right)^n f(x); p \right\} = (-1)^m P^n F(p) \text{ where } M \{f(x)\} = F(p).$$

8. Solve by using Laplace Transform :

$$(D^2 + 9)y = \cos 2t, \text{ if } y(0) = 1, y \left( \frac{\pi}{2} \right) = 1.$$

**Section – C**

**(Long Answer Questions)**

Note :- Answer any one question. Each answer should be given in 800 words. Each question carries 08 marks.

**1 x 8 = 8**

9. Prove that  $L \left[ \frac{\sin^2 t}{t} \right] = \frac{1}{4} \log \left( \frac{p^2+4}{p^2} \right)$  and deduce that :

(i)  $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt = \frac{1}{4} \log 5$

(ii)  $\int_0^{\infty} \frac{\sin^2 t}{t^2} dt = \frac{\pi}{2}$

10. (a) Find the solution of Diffusion equation  $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ ,  $x > 0, t > 0$  subject to the initial and boundary conditions  $u(x, 0) = 0, x > 0$ ;  $-K \left(\frac{\partial u}{\partial x}\right) = f(t)$  at  $x = 0, t > 0$  and  $u(x, t) \rightarrow 0$  as  $x \rightarrow \infty$  and  $t > 0$  where  $k$  &  $K$  are respectively the thermal diffusivity and conductivity of material of given solid.

(b) A flexible string has its end points on the x-axis at  $x = 0$  and  $x = c$ . At time  $t = 0$ , the string is given a shape defined by  $b \sin\left(\frac{\pi x}{c}\right)$ ,  $0 < x < c$  and released. Find the displacement of any point  $x$  of the string at any time  $t > 0$ .

**Internal Assignment-2014**

**Program Name M.Sc. / M.A. (Mathematics)**

**Paper Code – MT- 10(Mathematical Programming)**

**M.Sc. / M.A. (Final)**

**Max. Marks 20**

**Note:- The Internal Assignment has been divided into three sections A, B and C. Write Answer as per the given instructions .**

**Section –A**

**(Very Short Answer Type Questions)**

Note :- Answer all questions . As per the nature of the question you delimit your answer in one word , one sentence or maximum up to 30 words. Each question carries 1 (one ) mark.

**4 x1 = 4**

1. Define a convex set .
2. What do you mean by mixed Integer programming problem (Mixed I.P.P.)?
3. Define convex programming problem.
4. Write the Hessian matrix for the function  $f(x_1, x_2) = (x_1 - 2)^2 + (x_2 - 1)^2$

**Section – B**

**(Short Answer Questions)**

**Note :- Answer any two questions . Each answer should be given in 200 words. Each question carries 4 marks .**

**2 x4 = 8**

5. Find the nature of quadratic form  $Q(X) = X' AX$  where

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

6. Solve the integer programming problem :

Max  $z = 7x_1 + 9x_2$   
 S.t.  $-x_1 + 3x_2 \leq 6$   
 $7x_1 + x_2 \leq 3$   
 $x_1 \geq 0, x_2 \geq 0$   
 and  $x_1, x_2$  are integers

7. Use Lagrangian function to find the optimal solution of the following non linear programming problem:

$$\text{Minimize } f(X) = -3x_1^2 - 4x_2^2 - 5x_3^2$$

$$\text{Subject to } x_1 + x_2 + x_3 = 10$$

$$x_1, x_2, x_3 \geq 0$$

8. Use Wolfe's method to solve the following quadratic programming problem:

$$\text{Min. } f(x_1, x_2) = x_1^2 + x_2^2 - 2x_1 - 4x_2$$

$$\text{s.t. } x_1 + 4x_2 \leq 5$$

$$2x_1 + 3x_2 \leq 6$$

$$\& \quad x_1, x_2 \geq 0$$

### Section – C

#### (Long Answer Questions)

Note :- Answer any one question. Each answer should be given in 800 words. Each question carries 08 marks. **1 x 8 = 8**

9. Solve the following L.P.P. using dynamic programming:

$$\text{(a) Max } z = 3x_1 + 5x_2$$

$$\text{Such that } x_1 \leq 4$$

$$x_2 \leq 6$$

$$3x_1 + 2x_2 \leq 0$$

$$\& \quad x_1, x_2 \geq 0$$

$$\text{(b) Max. } z = 10x_1 + 30x_2$$

$$\text{S.T. } 3x_1 + 6x_2 \leq 168$$

$$12x_2 \leq 240$$

$$\& \quad x_1, x_2 \geq 0$$

- 10.(a) Explain the Kuhn-Tucker necessary conditions for the following problem

$$\text{Maximize } f(X)$$

$$\text{Subject to } g_i(X) \geq 0 \quad ; i = 1, 2, \dots, m$$

$$g_i(X) \leq 0, \quad i = m + 1, m + 2, \dots, P$$

$$h_i(X) = 0, \quad j = 1, 2, \dots, q$$

$$X_i \geq 0$$

- (b) Find an optimal solution of the following convex separable programming problem:

$$\text{Max } z = 3x_1 + 2x_2$$

$$\text{Subject to } 4x_1^2 + x_2^2 \leq 16$$

$$\& \quad x_1 x_2 \geq 0$$